

Chaos in daisyworld

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ABSTRACT

Lovelock proposed a concept, referred to as Gaia, in which feedbacks from the biosphere minimize fluctuation in climatic conditions. A simple model, referred to as daisyworld, was later developed to illustrate the Gaia concept. Daisyworld is defined on a cloudless flat or cylindrical planet with negligible atmospheric greenhouse gases in which bare soil and daisies of different colors interact so as to maintain stable climatic conditions. In the current paper, this daisyworld model is used to study the interaction between biota and their environment in more detail. It is found that periodic, and even chaotic, states can exist when the parameter controlling the feedback between biota and environmental temperature is changed. The existence of periodic and chaotic solutions is verified by their power spectra, fractal dimensions, and Lyapunov exponents. These results show that stable climatic conditions are not always maintained in daisyworld, despite the presence of daisies which supply the required feedback. While daisyworld is a simple model, the mathematical analysis of this model raises questions about the validity of the Gaia hypothesis.

1. Introduction

It is now widely accepted that the Earth is a single system which consists of the biota and their environment. These two elements of the system are closely coupled: the biota regulate the environment (e.g., climate on a planetary scale) and, in turn, the environment restricts the evolution of the biota and dictates what type of life can exist as a consequence of Darwinian natural selection. Changes in one part will influence the other, being opposed by negative feedback or enhanced by positive feedback, and this may lead to oscillation or chaos in the system. Therefore, in order for climate models to predict the consequences of changes caused by human activities (e.g., the increase of greenhouse gases), the biota should be included in the model. On the other hand, the biota and the environment of the Earth are so complex that this system, or even a single aspect of feedback in it, can not yet be adequately described by simple mathematical equations.

In the past few years, a model, daisyworld, which is an active system where the biota and the environment are tightly coupled, was first described by Lovelock (1982) and used to study the interaction between daisies of one or two species and the temperature of the environment (Watson and Lovelock, 1983). These authors found that the inclusion of feedback from the environment, regardless of its direction, stabilized daisyworld, and this was consistent with the earlier Gaia hypothesis that the climate and the chemical composition on the Earth have been and are maintained at a steady state by the presence of life itself (Lovelock and Margulis, 1974).

Though the feedback in daisyworld is much simpler than that on the Earth, research on such an imaginary planet may provide insight into the properties of the Earth's climate. Therefore, further studies are needed. The purpose of this paper is to study the fictional world in more detail, and, especially, to evaluate the chaotic properties of this model. Both qualitative and quantitative methods from modern chaos theory

have been utilized to verify the presence of chaos in daisyworld.

2. Description of the model for daisyworld

Daisyworld is a cloudless flat or cylindrical planet with negligible atmospheric greenhouse gases that bears life only in the form of different species of daisy. The governing equations used in this study are the same as in Watson and Lovelock (1983), but written in a more general way. A simple, realistic modification is also discussed at the end of this section.

The growth rate of the daisies is given by a set of equations in population ecology theory (Carter and Prince, 1981):

$$\frac{da_i}{dt} = a_i(x\beta_i - \gamma_i) \quad (i = 1, 2, \dots, m), \quad (1)$$

where a_i is the area covered by the i th species of daisy, which is measured as a fraction of the total surface area of daisyworld. The variable x is the unoccupied fertile area in which daisies could grow, i.e.,

$$x = p - \sum_{i=1}^m a_i, \quad (2)$$

where p is the total area of fertile ground. γ_i is the death rate per unit time and is taken to be the same constant for each species. β_i is the growth rate per unit time and per unit area, and it is assumed to be a parabolic function of the local temperature T_i :

$$\beta_i = C[1 - 0.003265(22.5 - T_i)^2], \quad (3)$$

which is zero at $T_i = 5^\circ\text{C}$ or 40°C , and reaches its maximum value of C (where C is a constant) at $T_i = 22.5^\circ\text{C}$. Watson and Lovelock (1983) restricted their attention to $C = 1$; in this paper, we consider the range $1 \leq C \leq 4$.

The effective temperature T_e ($^\circ\text{C}$) at which the planet's surface radiates can be obtained by the radiation budget equation on the surface:

$$\sigma(T_e + 273)^4 = SL(1 - A), \quad (4)$$

where σ is Stefan's constant, S is a constant having units of energy flux, L is a dimensionless measure of the luminosity of the sun of

daisyworld, and A is the average albedo of the planet, which is given by

$$A = (1 - p)A_{gs} + xA_{gr} + \sum_{i=1}^m a_i A_i, \quad (5)$$

where A_{gs} is the albedo of the sterile area in which the daisies can not grow, A_{gr} the albedo of the unoccupied fertile area, and A_i the albedo of the i th species.

The local temperature of different types of surface can be determined using conservation of energy balance of the planet. For the daisies, it can be expressed as

$$(T_i + 273)^4 = q(A - A_i) + (T_e + 273)^4, \quad (6)$$

where q is the conduction coefficient of solar energy among different types of surface. Eq. (6) can be simplified with small error for the temperatures of interest:

$$T_i = q'(A - A_i) + T_e, \quad (7)$$

where $q' = 0.25q/(295.5)^3$. Alternatively, for the particular value $q = SL/\sigma$, eq. (6) can be rewritten by means of eq. (4):

$$(T_i + 273)^4 = q(1 - A_i), \quad (8)$$

which means that the local temperatures are determined solely by the local radiation balance. Both of these conditions will be discussed below; however, most of our results will use the more realistic eq. (7).

To solve the closed set of equations (1)–(5) and (7) or (8) numerically, eq. (1) needs to be discretized. Using a forward-difference scheme, eq. (1) can be approximated by

$$a_i^{n+1} = a_i^n + \Delta t \left(\frac{da_i}{dt} \right)_n, \quad (9)$$

where Δt is the time step and implies, in physical terms, that there is a time delay in the interaction between the daisy population and the local temperatures. As a result, in order to accurately approximate eq. (1), we must choose Δt to be small. However, this allows the daisies to adjust to temperature variations instantaneously, which is unphysical. A more realistic model is to let Δt be the generation time, since this is the characteristic response time of the daisy population. Therefore, we will modify the above model by

replacing eq. (1) by eq. (9) with $\Delta t = 1$ (and with the generation time chosen as the unit of time). Note that a forward-difference scheme is chosen (as opposed to a centered-difference scheme), since this preserves the deterministic nature of daisyworld: the values of a_1^n, \dots, a_m^n determine the values of $a_1^{n+1}, \dots, a_m^{n+1}$.

It should be emphasized that the finite-difference equation (9) with $\Delta t = 1$ will yield different results from the differential equation (1). In the one- and two-species cases, chaos cannot occur for eq. (1). However, we believe that eq. (9) is a more realistic description of the interaction between daisies and their environment than is eq. (1), which implies an instantaneous feedback. Therefore, in this paper, we will use the governing equations (2)–(5), (7) or (8), and (9) with $\Delta t = 1$, although $\Delta t = 0.01$ is also utilized in Subsection 3.1 for comparison with the steady-state solutions obtained by Watson and Lovelock (1983) using the differential equation (1), since this time step is sufficiently small to approximate the instantaneous feedback assumed by them.

It is worth noting that, although a_i , x , and p in the governing equations are all expressed as fractions of the total area, the total growth rate $x\beta_i$ involves only a ratio of areas (cf. eq. (1)), and, thus, it is unaffected by this choice. It does, however, depend on the generation time, which has been chosen to be one. Therefore, varying β_i by varying C is related to varying this generation time. For one species, our equations are the same as those considered by May (1974), and he has shown that the important parameter is $\beta - \gamma$, which he takes as high as 5. Hence, we consider the ranges $0.3 \leq \gamma \leq 1$ and $1 \leq C \leq 4$ (and, thus, $\beta_i \leq 4$).

3. Chaotic behavior of daisyworld

As a nonlinear system with feedbacks between daisies and temperature, the governing equations of daisyworld could be expected to exhibit not only steady-state behavior, but also periodic, and even chaotic behavior. In the following, it is assumed that the planet's surface is totally fertile, i.e., $p = 1$. In addition, as in Watson and Lovelock (1983), S in eq. (4) is taken as 9.17×10^5 ergs $\text{cm}^{-2} \text{s}^{-1}$, and the albedo $A_g = A_{gr}$ in eq. (5) is adopted as 0.5.

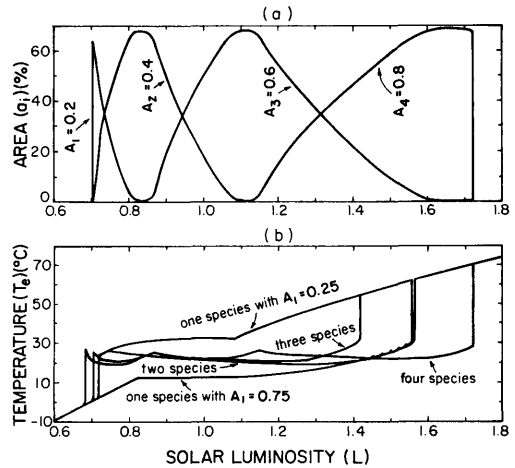


Fig. 1. Steady-state behavior of daisyworld. (a) Areas of four species, where A_1, A_2, A_3 , and A_4 are 0.2, 0.4, 0.6, and 0.8; (b) Effective temperatures for one to four species: for four species, albedos are as given above; for three species, A_1, A_2 , and A_3 are 0.1, 0.4, and 0.7; for two species, $A_1 = 0.75$ and $A_2 = 0.25$; for one species, $A_1 = 0.25$ or $A_1 = 0.75$.

3.1. Steady-state behavior of the system

Just as the Sun has been increasing in solar luminosity since the formation of the Solar System (Owen et al., 1979), the luminosity of the sun of daisyworld is assumed to rise slowly, so that, at each value of L , the possible steady state of daisyworld is evaluated by solving the governing equations forward in time with the initial values of different daisies set at the previous steady-state values, or 0.01 if these are zero.

Fig. 1 shows the steady-state behavior of four species of daisy and the temperature T_e as the luminosity L is incremented, where, for comparative purposes, the adopted parameters are the same as in Watson and Lovelock (1983) (i.e., $C = 1.0$, $\gamma_i = 0.3$, $q' = 20.0$, and $\Delta t = 0.01$). The effective temperatures for one to three species are also included in Fig. 1 for comparison.

It is seen from Fig. 1a that the area occupied by daisies is almost constant, i.e., $\sum_{i=1}^4 a_i \approx 0.7$, within the range of L from 0.7 to 1.7; however, as the luminosity increases, the species with lower albedos become extinct in the battle for survival of the fittest, and the species with the higher albedo dominates the planet. For each value of luminosity from 0.7 to 1.7, only one or two

species exist. Though not shown here, with only two or three species, daisyworld exhibits the same characteristics.

Fig. 1b shows that the temperature regulation of the planet is similar for two, three, or four species, and, therefore, for the remainder of this paper, we will consider only one or two species for simplicity. One species, with $A_1 = 0.25$, will be referred to as “black”, and the other, $A_2 = 0.75$, will be referred to as “white” to emphasize the contrast, though neither of them is perfectly black or white. Hereafter, subscripts b and w will be used to represent black and white daisies for simplicity.

When eq. (8) is used to compute T_i , instead of eq. (7), i.e., $q = SL/\sigma$, the steady-state behavior for two species is shown in Fig. 2. It is found that, for the range of L from 0.8 to 1.5, both black and white daisies are eliminated, and the temperature increases almost linearly. Computations also show that the daisyworld model for one or two species using eq. (8) exhibits no hysteresis when the luminosity decreases; whereas, the system using eq. (7) does (see Fig. 1c in Watson and Lovelock, 1983). Therefore, unlike eq. (7), eq. (8) does not yield a self-regulating system.

Calculations also show that the steady-state behavior is the same for time steps Δt of 0.01, 0.5, or 1.0, although the time needed to obtain the

steady state is different. Furthermore, it is found that, when steady-state behavior is reached, it is similar for different parameter values, e.g., changing q' from 20 to 40 or 60, or changing C in eq. (3) from 1 to 2.

3.2. Chaos in daisyworld

Parameters needed in this subsection besides p , S , and A_g are taken as follows: $\Delta t = 1.0$, $q' = 20$ with eq. (7), $A_b = 0.25$ for black daisies and $A_w = 0.75$ for white daisies.

During a stage when the luminosity is fixed with time in daisyworld, we alter the internal environment by increasing the interaction between the daisies and the environmental effective temperature T_e , and by increasing the death rate; i.e., the parameters C in eq. (3) and γ_i in eq. (1) are increased. In this case, chaos may appear, and we present such numerical results in Subsection 3.2.2. First, however, we present an analytic analysis.

3.2.1. Theoretical analysis of the chaotic regime in the daisyworld model

When eq. (8) is used in the model, constant luminosity leads to constant T_i in eq. (8) and, hence, constant β_i from eq. (3). The difference equation (9) with the above constants can be written as

$$a_i^{n+1} = (1 + \beta_i - \gamma_i) a_i^n - \beta_i a_i^n \sum_{j=1}^m a_j^n$$

$$(i = 1, 2, \dots, m), \tag{10}$$

where β_i and γ_i are constants.

For one species, we let

$$y = \frac{\beta}{1 + \beta - \gamma} a, \tag{11}$$

so that eq. (10) may be rewritten as

$$y^{n+1} = (1 + \beta - \gamma) y^n (1 - y^n), \tag{12}$$

which is the well known logistic difference equation (see, e.g., May (1976) among others). The solution y^n of this equation is chaotic for many values of $1 + \beta - \gamma$ in the range

$$3.57 \leq 1 + \beta - \gamma \leq 4. \tag{13}$$

On the other hand, the physical restriction on a is $0 \leq a \leq 1$, and the maximum value of y in eq. (12) can be easily obtained as $(1 + \beta - \gamma)/4$.

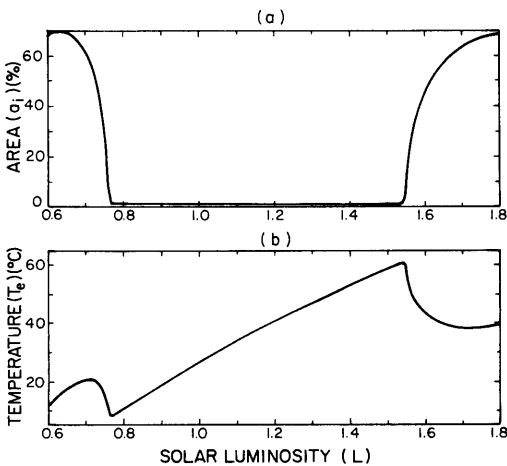


Fig. 2. Steady state responses of daisyworld with eq. (8). (a) Areas of black and white daisies; (b) Effective temperature T_e .

Therefore, eq. (11) yields the condition

$$(1 - \sqrt{\gamma})^2 \leq \beta \leq (1 + \sqrt{\gamma})^2. \tag{14}$$

Combining this with eq. (13) yields

$$\begin{aligned} \max[(1 - \sqrt{\gamma})^2, (2.57 + \gamma)] &\leq \beta \\ &\leq \min[(3 + \gamma), (1 + \sqrt{\gamma})^2], \end{aligned} \tag{15}$$

and, for $\gamma \leq 1$, this becomes

$$2.57 + \gamma \leq \beta \leq (1 + \sqrt{\gamma})^2, \tag{16}$$

which also implies that

$$0.616 \leq \gamma \leq 1. \tag{17}$$

For more than one species, with $\gamma_i = \gamma$ (i.e., the same for each species), we let

$$a = \sum_{i=1}^m a_i, \quad \beta = \frac{1}{a} \sum_{i=1}^m \beta_i a_i, \quad Y = \frac{\beta}{1 + \beta - \gamma} a, \tag{18}$$

and we sum eq. (10) over i to yield

$$Y^{n+1} = (1 + \beta - \gamma) Y^n (1 - Y^n), \tag{19}$$

which is the same as eq. (12). However, while eq. (19) indicates that chaos should again be expected, the specific results for the case with one species are not applicable, since β now depends on n . If, instead of eq. (8), we use the more realistic eq. (7), then β_i is no longer independent of n . However, the analysis leading to eqs. (12) and (19) is still valid, with β now dependent on n in both cases. Therefore, chaos is again expected to occur, although we can give no quantitative prediction of the parameter regime. In the numerical results which follow, we use eq. (7), rather than eq. (8), since only the former can lead to self-regulation (cf. Figs. 1b and 2b).

3.2.2. Numerical results on chaos in daisyworld

Case 1. Daisyworld with only black daisies

Figs. 3 and 4 show a chaotic state of the planet using eq. (7) with only black daisies at $L = 0.90$, $C = 4.0$, and different values of γ_b . Other parameters are as previously specified. The plots are generated by solving the governing equations with the initial value of a_b (the area covered by black daisies) set at the steady-state value in Subsection 3.1.

It is obvious in Figs. 3 and 4 that black daisies alone can adjust to drastic change in the environmental temperature T_e (as large as 19°C)

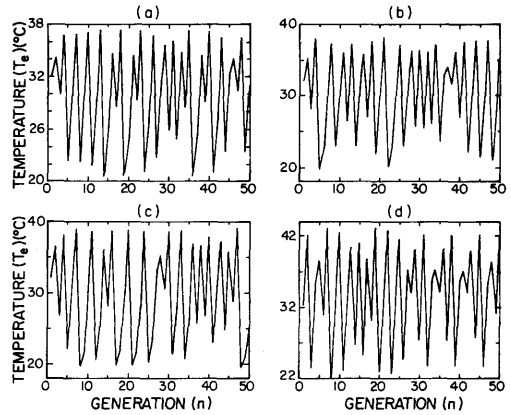


Fig. 3. Chaotic behavior of the effective temperature T_e in daisyworld with only black daisies at $L = 0.90$ and $C = 4.0$. (a) $\gamma_b = 1.0$; (b) $\gamma_b = 0.9$; (c) $\gamma_b = 0.8$; (d) $\gamma_b = 0.3$.

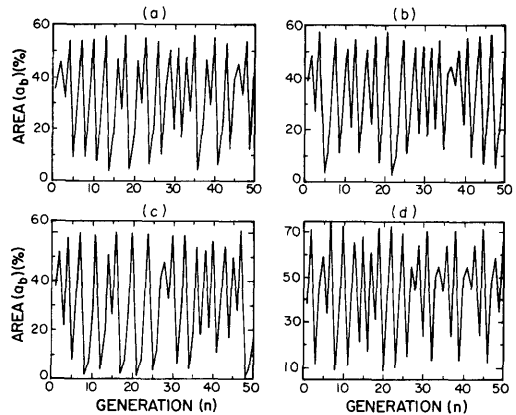


Fig. 4. The same as in Fig. 3, except for the area a_b occupied by black daisies.

without becoming extinct. However, homeostasis is not maintained; instead, the effective temperature and the area covered by black daisies is chaotic and unpredictable. It is also interesting to note that variations of a_b and T_e in Figs. 3 and 4 are quite similar (cf., e.g., Fig. 3a and Fig. 4a). This feature is explained in the appendix.

According to eqs. (16) and (17), no chaos occurs in the logistic difference equation when $\gamma = 0.3$ and $\beta = \text{constant}$. However, using eq. (7), in which case, $\beta \neq \text{constant}$, daisyworld is chaotic at $\gamma_b = 0.3$, which shows that the inclusion of the feedback from the environmental temperature

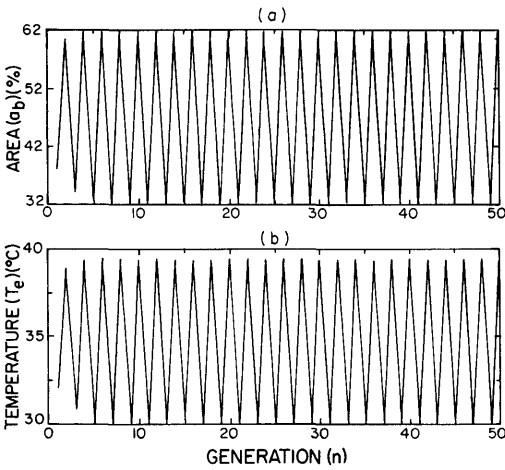


Fig. 5. Values of a_b and T_e in the model with only black daisies at $L = 0.90$, $C = 3.0$, and $\gamma_b = 0.3$. (a) a_b ; (b) T_e .

widens the chaotic regime of daisyworld. This also illustrates the well known fact that, once the coefficient in the logistic map given by eq. (12) becomes variable, it is difficult to analytically predict the occurrence of chaos from the equation.

The occurrence of chaos in the model is sensitive to the value of C , though not sensitive to the value of γ_b in this case. Fig. 5 shows the results for $C = 3.0$ and $\gamma_b = 0.3$. We see from Fig. 5 that both T_e and a_b are in stable cycles of period 2. The amplitude of the oscillation is 10°C for T_e and 32% for a_b . Both amplitudes are smaller than those for the chaotic states. When $C = 3.0$ and $\gamma_b = 0.8, 0.9$, or 1.0 , periodic solutions are also obtained, and the amplitudes of a_b and T_e decrease with the increase of γ_b . At $C = 2.0$, the daisyworld model becomes steady-state for $\gamma_b = 0.3, 0.8, 0.9$, or 1.0 . Chaos occurs with $C = 4.0$ for a wide range of values of the luminosity.

Case 2. Daisyworld with only white daisies

The results with $L = 0.92$ and $C = 4.0$ are shown in Fig. 6. At $\gamma_w = 1.0$, the environmental temperature and the area occupied by white daisies appear to oscillate periodically with a period of 10. However, as shown in the next section, they are, in fact, slightly chaotic. When γ_w is changed from 1.0 to 0.8, the apparent periodicity disappears and the chaos is apparent. The largest amplitudes of the variations of T_e and

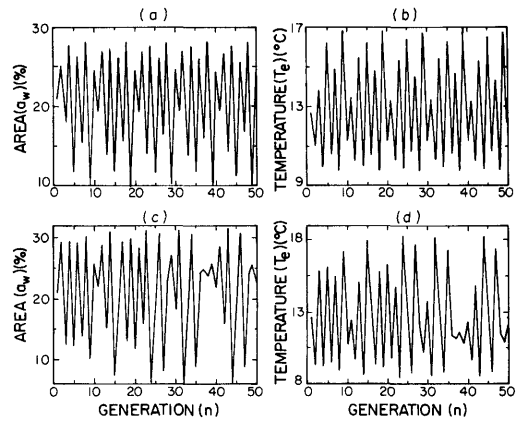


Fig. 6. Chaotic behavior of a_w and T_e in daisyworld with only white daisies at $L = 0.92$, $C = 4.0$. (a) a_w with $\gamma_w = 1.0$; (b) T_e with $\gamma_w = 1.0$; (c) a_w with $\gamma_w = 0.8$; (d) T_e with $\gamma_w = 0.8$.

a_w are roughly 7°C and 18% for $\gamma_w = 1.0$, and roughly 10°C and 25% for $\gamma_w = 0.8$. It is also found that, at $\gamma_w = 0.3$, the white daisies become extinct by the fourth generation.

In this case, the variations of a_w and T_e in Fig. 6 have opposite phase and are inverted versions of each other (cf. Figs. 6a and b; Figs. 6c and d), which is different than in Case 1 for black daisies. The appendix gives the explanation for this difference.

Computations with white daisies also show that T_e and a_w oscillate with period 2 at $C = 3.0$, $\gamma_w = 0.3$, and are in steady-state at $C = 3.0$, $\gamma_w = 0.8, 0.9$, or 1.0 , and at $C = 2.0$, $\gamma_w = 0.3, 0.8, 0.9$, or 1.0 . Finally, for $C = 4.0$, the range of luminosities giving rise to chaos is smaller for white daisies than for black daisies. Unlike Case 1, the occurrence of chaos is sensitive to both C and γ_w in this case.

Case 3. Daisyworld with both black and white daisies

Chaos occurs in daisyworld not only with one species, but also with two species. Fig. 7 shows the chaotic state of the model for $L = 0.80$ and $C = 4.0$. The largest amplitudes of variations of a_w , a_b , and T_e are roughly 12% , 80% , and 23°C , respectively. At $C = 3.0$, a_w , a_b , and T_e in daisyworld vary in stable cycles of period 2 for $\gamma_b = \gamma_w = 0.8, 0.9$, or 1.0 . The periodic state for $\gamma_b = \gamma_w = 0.8$ is shown in Fig. 8. It is seen that a_w , a_b , and T_e all increase or decrease at the same

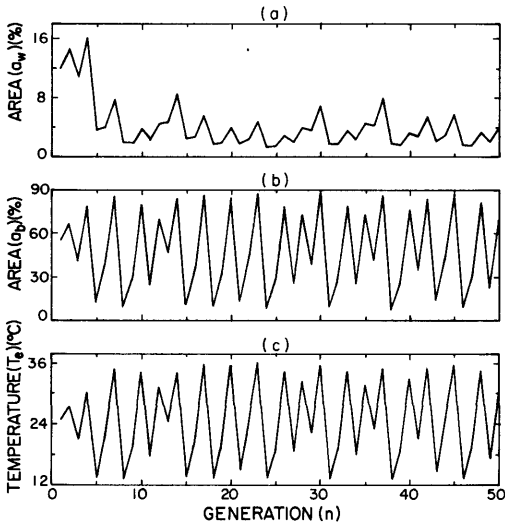


Fig. 7. Chaotic behavior of a_w , a_b , and T_e in daisyworld with black and white daisies at $L = 0.80$, $C = 4.0$. (a) a_w with $\gamma_w = \gamma_b = 1.0$; (b) a_b with $\gamma_w = \gamma_b = 1.0$; (c) T_e with $\gamma_w = \gamma_b = 1.0$.

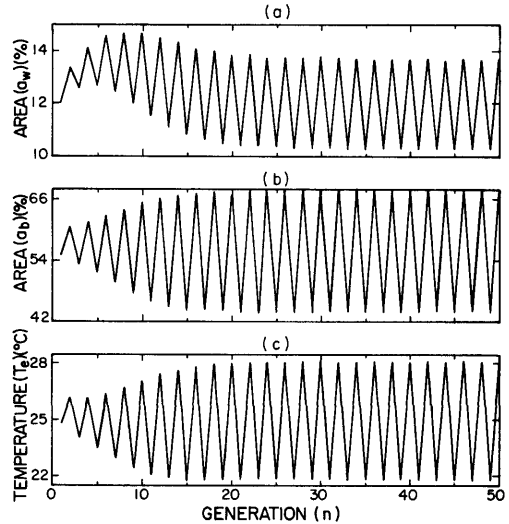


Fig. 8. Periodic behavior of daisyworld with two species at $L = 0.80$, $C = 3.0$, and $\gamma_b = \gamma_w = 0.8$. (a) a_w ; (b) a_b ; (c) T_e .

time. The largest amplitudes of variations for a_w , a_b , and T_e are roughly 4%, 24%, and 7°C, respectively. At $C = 2.0$, the homeostasis is maintained in the model for $\gamma_b = \gamma_w = 0.3, 0.8, 0.9$, or 1.0, just as with only one species. It is also found that variations of T_e and a_b have the same phase, but they generally have no phase relationship with a_w for the parameters used in Figs. 7 and 8 (cf. Fig. 7). This is discussed in the appendix.

The above results for one and two species show that the coupling strength C is the most important parameter in determining the qualitative behavior of daisyworld. As C is increased from 1.0 to 4.0, the behavior of daisyworld will change from a steady state to periodicity to chaos over a wide range of values of luminosity, and, thus, the system will become extremely sensitive to minor changes in the initial values of a_w and a_b . As a result, homeostasis can not be maintained by the presence of one or two species on the planet. Furthermore, since the chaos is more pronounced with two species than with one, there is no reason to believe that it will be eliminated by including additional species (also, recall Fig. 1, which showed that only one or two species at a time

were involved in maintaining the steady-state behavior). Therefore, the conclusion of Watson and Lovelock (1983), based on the differential equation (1), that daisyworld always shows greater stability with daisies present, is not a general result, and the remarks made by Lovelock (1986), that the inclusion of feedback from the environment appears to stabilize the system, are also not true in general.

4. Verification of chaos in daisyworld

In this section, we verify the existence of chaos in daisyworld by examining power spectra and computing fractal dimensions and Lyapunov exponents. The analysis of power spectra is often used to distinguish, qualitatively, quasiperiodic or chaotic behavior from periodic structure, and to identify different periods embedded in a chaotic signal. Chaos is characterized by the presence of broadband noise in the power spectrum. In daisyworld, the discrete time series of the effective temperature T_e , consisting of 2000 points, is used to compute the spectra by means of a fast Fourier transformation (FFT). The time series for the areas covered by black or white daisies give similar results.

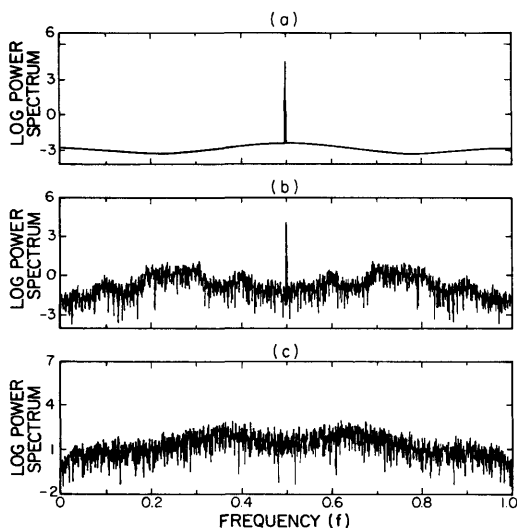


Fig. 9. Power spectra for the cases of one and two species in daisyworld. (a) Periodic state of black daisies for the parameters used to create Fig. 5. (b) Slightly chaotic state of white daisies with $\gamma_w = 1.0$. Other parameters are as used to generate Fig. 6. (c) Chaotic state of white and black daisies with $\gamma_b = \gamma_w = 1.0$. Other parameters are as used to create Fig. 7.

When the power spectra are computed for the cases discussed in Section 3, we find broadband noise for the chaotic behavior in Figs. 3, 4, 6, and 7, and sharp peaks for the periodic behavior in Fig. 5 and Fig. 8. Some examples are shown in Fig. 9. In particular, for daisyworld with only white species and $\gamma_w = 1.0$, the power spectrum shows unambiguously that the behavior is slightly chaotic (note the low noise level), even though this is not apparent from Figs. 6a and 6b.

An important feature of dissipative dynamical

systems which exhibit chaotic behavior is that trajectories in phase space approach a strange (i.e., fractal) attractor (see, e.g., Lorenz, 1963). The key features of a strange attractor are a fractal dimension and sensitivity to initial conditions (i.e., the separation of neighboring trajectories exponentially in time) (Bergé et al., 1984). The quantitative measure of this divergence of trajectories is the Lyapunov exponent (which is sometimes called the Lyapunov number).

The fractal dimension ν given by Grassberger and Procaccia (1983), which is the correlation dimension, is used in this study (rather than the Hausdorff dimension D), since the computation of the correlation dimension is relatively easy. The correlation dimension ν provides a rigorous lower bound to D , and the two are generally close in value.

Starting with the discrete time series of the effective temperature T_e , a k -dimensional phase space is then constructed by using the vector $x_i = (T_e(i), T_e(i + 1), \dots, T_e(i + k - 1))$, where $T_e(i)$ is the value of T_e at the i th generation. Then a spatial correlation function can be expressed as (Grassberger and Procaccia, 1983)

$$C(r) = \lim_{m \rightarrow \infty} \frac{1}{m^2} \sum_{i,j=1}^m H(r - |x_i - x_j|) \sim r^\nu, \quad (20)$$

where H is the Heaviside function defined by $H(x) = 1$ for positive x , and $H(x) = 0$, otherwise. As $k \rightarrow \infty$, ν approaches a value independent of k (usually $k > 2\nu$ is sufficient), and this value is the correlation dimension. Our calculations show that a time series of 600 values is sufficient to obtain ν with reasonable precision. The correlation dimensions for some cases of chaos in Section 3 are given in Table 1. It is seen that, in

Table 1. Correlation dimension ν and Lyapunov exponents λ for some cases of chaos from Section 3

	Black daisies only (parameters as in Fig. 3)				White daisies only (parameters as in Fig. 6)		Black and White daisies (parameters as in Fig. 7)
	$\gamma_b = 1.0$	$\gamma_b = 0.9$	$\gamma_b = 0.8$	$\gamma_b = 0.3$	$\gamma_w = 1.0$	$\gamma_w = 0.8$	$\gamma_w = \gamma_b = 1.0$
ν	0.90	0.98	0.97	0.95	0.81	0.91	1.9
λ_1	0.51	0.58	0.60	0.44	0.18	0.42	0.47
λ_2	—	—	—	—	—	—	-0.22

each case, the correlation dimension is fractional, which is one characteristic of chaos.

Next, we examine the Lyapunov exponents. For only a single species of daisy, we choose an initial value x_0 for the area covered by daisies and iterate it to generate the time series x_0, x_1, x_2, \dots . We then choose a slightly different initial condition x'_0 and generate another time series x'_0, x'_1, x'_2, \dots . When chaos is present, the separation $\Delta x_n = |x'_n - x_n|$ will grow exponentially with increasing n , and this growth rate is described by the Lyapunov exponent

$$\lambda = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \frac{\Delta x_n}{\Delta x_0} \quad (21)$$

As it is written, eq. (21) is only valid in the limit as $\Delta x_0 \rightarrow 0$. For finite Δx_0 , the argument of the limit approaches the desired value as n increases, but, when n becomes too large, it begins to fluctuate randomly. Thus, n must not be allowed to get too large (for $\Delta x_0 = 10^{-8}$, $n = 40$ gives a reasonable value for λ).

For two species of daisy, we consider the vector $\mathbf{x} = (a_w, a_b)$. Choosing three nearby initial conditions, \mathbf{x}_0 , $\mathbf{x}_0^{(1)}$, and $\mathbf{x}_0^{(2)}$, we generate three time series and consider the separation vectors $\Delta \mathbf{x}_n^{(1)} = \mathbf{x}_n^{(1)} - \mathbf{x}_n^{(1)}$ and $\Delta \mathbf{x}_n^{(2)} = \mathbf{x}_n^{(2)} - \mathbf{x}_0^{(2)}$. Since there are now two species of daisy, there are two Lyapunov exponents, and the larger exponent λ_1 is given by eq. (21) using the length of either $\Delta \mathbf{x}_n^{(1)}$ or $\Delta \mathbf{x}_n^{(2)}$. The smaller exponent λ_2 is then given by

$$\lambda_1 + \lambda_2 = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \frac{S_n}{S_0}, \quad (22)$$

where S_n is the area of the parallelogram determined by the two vectors $\Delta \mathbf{x}_n^{(1)}$ and $\Delta \mathbf{x}_n^{(2)}$.

It should be noted that eqs. (21) and (22) give the Lyapunov exponents at the point \mathbf{x}_0 . To obtain the global exponents, we average these results over many initial conditions \mathbf{x}_0 chosen along some trajectory. For a further discussion of Lyapunov exponents, see Bennetin et al. (1976).

Lyapunov exponents are also included in Table 1 (for the cases of only one species, there is only one Lyapunov exponent). We see that the exponent λ_1 is positive in all cases, which shows unambiguously that chaos is present.

In summary, by analyzing the power spectra, fractal dimensions, and Lyapunov exponents, we have verified the existence of chaos in daisyworld with one or two species.

5. Conclusions and comments

The simple daisyworld model has been used to study the interaction between biota and their environment. It has been found that periodic, and even chaotic, behavior exists in daisyworld with one or two species, with the controlling parameter being the strength of the coupling between the daisies and the environmental temperature. Computations of the power spectra, fractal dimensions, and Lyapunov exponents verify the existence of periodic and chaotic behaviors in the model.

These results show that daisyworld is not always in a steady state as predicted by the Gaia hypothesis; instead, the state of daisyworld can show extreme sensitivity to minor fluctuations in the effective temperature or the areas covered by daisies when in its chaotic regime. Therefore, the presence of daisies on the imaginary planet does not always stabilize the climate conditions of the environment, and the inclusion of negative feedback from the environment does not always lead to steady-state behavior.

Though only daisies are present and spherical geometry is not considered in the simple daisyworld model, the interaction between biota and their environment is analogous to the more complex feedbacks on the earth, including greenhouse gases, land use, and cloud and global climate interactions. With the increasing concern for the environment of the Earth where we live, more complete coupled modeling of the interaction of climate with the biosphere and the lithosphere on the Earth, including the existence of chaotic states, needs to be developed, and this will be the subject of future work.

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7. Appendix

For one species with the parameters given in the paper, eqs. (4) and (5) yield

$$(T_e + 273)^4 = \frac{SL}{\sigma} [0.5 + (0.5 - A_i) a_i], \quad (\text{A1})$$

and, since $5 \leq T_e \leq 40$, we have

$$(T_e + 273)^4 \cong (273)^4 + 4(273)^3 T_e. \quad (\text{A2})$$

For black daisies with $A_b = 0.25$, eq. (A1) becomes

$$(T_e + 273)^4 = \frac{SL}{4\sigma} (2 + a_b), \quad (\text{A3})$$

so that T_e and a_w oscillate in phase. On the other

hand, for white daisies with $A_w = 0.75$, we have

$$(T_e + 273)^4 = \frac{SL}{4\sigma} (2 - a_w), \quad (\text{A4})$$

and T_e and a_w have opposite phase (hence, in Fig. 6, the plots of T_e and a_w are inverted versions of each other).

For both black and white daisies with $A_w = 0.75$ and $A_b = 0.25$, we can obtain from eqs. (4) and (5) that

$$(T_e + 273)^4 = \frac{SL}{4\sigma} (2 + a_b - a_w). \quad (\text{A5})$$

Since the variation of a_b is much larger than that of a_w (cf. Figs. 7 and 8), T_e and a_b have the same phase. However, no such phase relationship need exist between a_w and either T_e or a_b .

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