A Comparison of Three-Dimensional and Two-Dimensional Numerical Predictions of Sea Breezes

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ABSTRACT

Two-dimensional and three-dimensional sea breeze model results are compared for the same set of initial conditions and values of prescribed parameters. It is shown that with two-dimensional forcing, the two models produce identical results, but only when the explicit horizontal diffusion in the two-dimensional model is increased so as to account for resolvable scale fluxes which cannot be properly handled without including the third dimension. It is additionally demonstrated that a two-dimensional sea breeze model cannot produce accurate simulations of the sea breeze over south Florida even with added vertical resolution. The conclusion is made that for most practical simulations of the sea breeze, a full three-dimensional model is required.

1. Introduction

The purpose of this paper is to examine the utility of two-dimensional numerical simulations of the sea breeze which, of course, develops in a fully three-dimensional atmosphere. In most previous two-dimensional models of the sea breeze, the variations of the dependent variables normal to the plane of consideration are ignored, although a lateral component of velocity is generally allowed to develop due to the Coriolis effect and through turbulent diffusion. The recent models of Estoque (1961, 1962), Moroz (1967) and Neumann and Mahrer (1971), for example, were of this form. The limitation of the calculations to two dimensions has not been made by choice, but rather has been dictated by the computational limitations of the computer which, until recently, has been insufficient to handle the large number of data points required. Even today a fully three-dimensional sea breeze simulation takes a substantial amount of computer time, thus making extensive experimentation with such a model prohibitively expensive.

In this paper we will examine the ability of a two-dimensional model to adequately simulate a three-dimensional sea breeze flow. We will do this by comparing a two- and a three-dimensional numerical model which are identical except that the two-dimensional model neglects gradients of the dependent variables parallel to the coastline. Comparison experiments will first be discussed where the three-dimensional model is run with a simulated two-dimensional forcing so that the two-dimensional model is compared in the best possible light. Then, the two-dimensional model will be integrated over a cross section of south Florida where the two-dimensional approximation would be expected to be best satisfied. The results will be compared against the full three-dimensional calculation where the curvature of the Florida coastline and Lake Okeechobee were included in the calculations.

The experiments should help answer the questions:

1) Does the two-dimensional model produce the same solution as the three-dimensional model when the forcing is two dimensional?

2) Under realistic situations, does the two-dimensional model produce accurate predictions of the sea breeze, or will we inevitably be forced to use the more costly and cumbersome three-dimensional model?

2. Equations

The three-dimensional model has been discussed elsewhere in detail by Pielke (1974) and has demonstrated a remarkable ability to predict the locations and movement of the preferred regions of thunderstorm activity over south Florida on undisturbed days as a function of the large-scale flow. A two-dimensional model was created from this three-dimensional model by neglecting the north-south gradients of the dependent variables, since the south Florida coastline is oriented in roughly a north-south direction. Otherwise, the two models are identical.

Since the derivations of the equations for the three-dimensional model are presented in Pielke (1974), only the forms of the equations are presented here. The underlined terms are those which are neglected in the two-dimensional version of the model. The equations

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are given by

$$\frac{\partial\tilde{u}}{\partial t}=-(\hat{u}+\tilde{u})\frac{\partial\tilde{u}}{\partial x}-(\hat{v}+\tilde{v})\frac{\partial\tilde{u}}{\partial y}-\tilde{w}\frac{\partial}{\partial z}(\tilde{u}+\hat{u})-\hat{\theta}\frac{\partial\tilde{\Pi}}{\partial x}-fv_g\frac{\tilde{\theta}}{\hat{\theta}}+f\tilde{v}-\hat{f}\tilde{w}+\frac{\partial}{\partial x}K_H\frac{\partial\tilde{u}}{\partial x}+\frac{\partial}{\partial y}K_H\frac{\partial\tilde{u}}{\partial y}+\frac{\partial}{\partial z}[\hat{K}_z^{(m)}+\tilde{K}_z^{(m)}]\frac{\partial\tilde{u}}{\partial z}+\frac{\partial}{\partial z}\tilde{K}_z^{(m)}\frac{\partial\hat{u}}{\partial z},\quad(1)$$

$$\frac{\partial\tilde{v}}{\partial t}=-(\hat{u}+\tilde{u})\frac{\partial\tilde{v}}{\partial x}-(\hat{v}+\tilde{v})\frac{\partial v}{\partial y}-\tilde{w}\frac{\partial}{\partial z}(\tilde{v}+\hat{v})-\hat{\theta}\frac{\partial\tilde{\Pi}}{\partial y}+fu_g\frac{\tilde{\theta}}{\hat{\theta}}-f\tilde{u}+\frac{\partial}{\partial x}K_H\frac{\partial\tilde{v}}{\partial x}+\frac{\partial}{\partial y}K_H\frac{\partial\tilde{v}}{\partial y}+\frac{\partial}{\partial z}[\hat{K}_z^{(m)}+\tilde{K}_z^{(m)}]\frac{\partial\tilde{v}}{\partial z}+\frac{\partial}{\partial z}\tilde{K}_z^{(m)}\frac{\partial\hat{v}}{\partial z},\quad(2)$$

$$\frac{\partial\tilde{\theta}}{\partial t}=-(\hat{u}+\tilde{u})\frac{\partial\tilde{\theta}}{\partial x}-(\hat{v}+\tilde{v})\frac{\partial\tilde{\theta}}{\partial y}-\tilde{w}\frac{\partial}{\partial z}(\tilde{\theta}+\hat{\theta})+\frac{\partial}{\partial x}K_H\frac{\partial\tilde{\theta}}{\partial x}+\frac{\partial}{\partial y}K_H\frac{\partial\tilde{\theta}}{\partial y}+\frac{\partial}{\partial z}[K_z^{(\theta)}+\tilde{K}_z^{(\theta)}]\frac{\partial\tilde{\theta}}{\partial z}+\frac{\partial}{\partial z}\tilde{K}_z^{(\theta)}\frac{\partial\hat{\theta}}{\partial z},\quad(3)$$

$$\frac{\partial\tilde{w}}{\partial z}=-\left(\frac{\partial\tilde{u}}{\partial x}+\frac{\partial\tilde{v}}{\partial y}\right),\quad(4)$$

$$\frac{\partial\tilde{\Pi}}{\partial z}=g\frac{\tilde{\theta}}{\hat{\theta}^2},\quad(5)$$

where the terms with the caret are synoptic-scale variables while those with the tilde are grid-volume averaged perturbations from the synoptic state (a complete list is given in the Appendix). Since the simplifications and the assumptions used in the model are discussed in Pielke (1973), they will be only briefly summarized here.

The synoptic-scale velocity is calculated at the initial times as a balance of the pressure gradient, Coriolis and eddy friction forces in a neutrally stratified boundary layer over a horizontally homogeneous water surface, while the synoptic-scale potential temperature stratification is arbitrarily specified. The horizontal gradients of the synoptic-scale variables, except pressure, and the synoptic-scale vertical motion are assumed identically zero.

The grid-volume averaged perturbations from the synoptic state are determined from the time-dependent solution of a semi-implicit forward-upstream finite-difference analog to Eqs. (1)–(5). The subgrid-scale fluxes are parameterized using vertical exchange coefficients based on a conglomeration of work by Yamamoto and Shimanuki (1966), Blackadar and Tennekes (1968), O'Brien (1970) and Clark (1970), and horizontal exchange coefficients based on the work of Leith (1969).

3. Discussion of the horizontal exchange coefficient

The magnitudes of the parameterized form of the horizontal diffusion will be shown to have a significant influence on the degree of agreement between the two- and three-dimensional models. In the three-dimensional model, the horizontal exchange coefficient is of the form

$$K_H=\alpha_{3D}(\Delta X)(\Delta Y)\left\{\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2+\frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]\right\}^{\frac{1}{2}},\quad(6)$$

where α_{3D} is a constant determined by numerical experiment, while ΔX and ΔY are the respective grid spacings in the x and y directions. The form of (6), developed by Leith (1969), is based on the assumption that the grid scale lies within an inertial range of three-dimensional homogeneous isotropic turbulence. Although this is not true in any existing sea breeze model, this form of the exchange coefficient results in quite reasonable solutions when the arbitrary constant α_{3D} is adjusted so as to just eliminate horizontal irregularities in the shortest wavelengths resolved by the model, but not to smear the fields of the dependent variables excessively.

In the two-dimensional model, two parameterized forms of the horizontal exchange coefficient were tried. In the first form, the y-derivative terms in (6) were eliminated so that the coefficient is of the form

$$K_H=\alpha_{2D}(\Delta X)^2\left[\left(\frac{\partial v}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2\right]^{\frac{1}{2}}.\quad(7)$$

Leith, however, claimed that because vorticity is conserved in two-dimensional flows, these flows have different statistical properties than three-dimensional flows. As a result of this conservation property, and because contrary to the three-dimensional case, kinetic energy is transferred to larger scales, Leith proposed that the two-dimensional eddy viscosity coefficient be a function of the gradient of vorticity rather than the deformation. Lilly (1969) proposed a similar form of the exchange coefficient in his two-dimensional numerical simulation of turbulence. Therefore, a second form of the horizontal exchange coefficient used in the two-dimensional model is given by

$$K_H=\alpha'_{2D}(\Delta X)^3\left|\frac{\partial\xi}{\partial x}\right|, \tag{8}$$

where the x-z component of vorticity is given by

$$\xi=\frac{\partial w}{\partial x}-\frac{\partial u}{\partial z}.$$

4. Specification of the experiments

Perturbation motion is initiated in the models by prescribing a time-dependent surface heating which, by turbulent momentum and heat transfer, alters the equilibrium synoptic-scale flow causing the sea breeze circulations.

In the experiments presented below, and in both models, the following conditions are identical:

$f=6.38\times10^{-5}$ sec^{-1}	$\hat{\theta}(t=0,\ 1.52\text{ km})=303$K
$g=980$ cm sec^{-1}	$\hat{\theta}(t=0,\ 2.12\text{ km})=305.5$K
$\hat{\theta}(t=0,\ \text{surface})=298$K	$\hat{\theta}(t=0,\ 3.02\text{ km})=310$K
$\hat{\theta}(t=0,\ 75\text{ m})=298$K	$\hat{\theta}(t=0,\ 4.22\text{ km})=317$K
$\hat{\theta}(t=0,\ 660\text{ m})=300$K	$\hat{\theta}(t=0,\ 4.82\text{ km})=320.5$K.

The surface roughness length over land, required in the parameterization scheme for the vertical turbulence diffusion, is assumed as 4 cm, while the surface temperature over land is given as a sinusoidal heating function with a half period of 13 hr and 44 min and a maximum amplitude of 308K.

There are eight levels in the vertical and 33 grid points in the x direction given by

$$z(j)=\begin{cases} j\cdot 50\text{ m}, & 1\leqslant j\leqslant 2\\ 1.22\text{ km}+(j-3)\cdot 600\text{ m}, & 3\leqslant j\leqslant 5\\ 3.62\text{ km}, & j=6\end{cases}$$

where the top is a material surface with an initial value of 4.82 km and

$$x(i)=\begin{cases} x(i-1)+55\text{ km}-(i-2)\cdot 11\text{ km}, & 2\leqslant i\leqslant 5\\ x(i-1)+11\text{ km}, & 6\leqslant i\leqslant 29\\ x(i-1)+55\text{ km}-(33-i)\cdot 11\text{ km}, & 30\leqslant i\leqslant 33\end{cases}$$

In the three-dimensional model there are 36 grid points in the y-direction given by

$$y(k)=\begin{cases} y(k-1)+55\text{ km}-(k-2)\cdot 11\text{ km}, & 2\leqslant k\leqslant 5\\ y(k-1)+11\text{ km}, & 6\leqslant k\leqslant 32\\ y(k-1)+55\text{ km}-(36-k)\cdot 11\text{ km}, & 33\leqslant k\leqslant 36\end{cases}$$

In the next two sections, two experiments will be discussed—one in which the two models are compared for an extended north-south landstrip with the geostrophic wind from the east (hereafter called the rectangular sea breeze experiment), and one in which solutions from the two models are compared for a cross section of south Florida where the coastline is oriented in approximately a north-south direction and where the curvature of the coastline is small.

The grid mesh and outline of land mass used in the three-dimensional experiments is given in Fig. 1. The

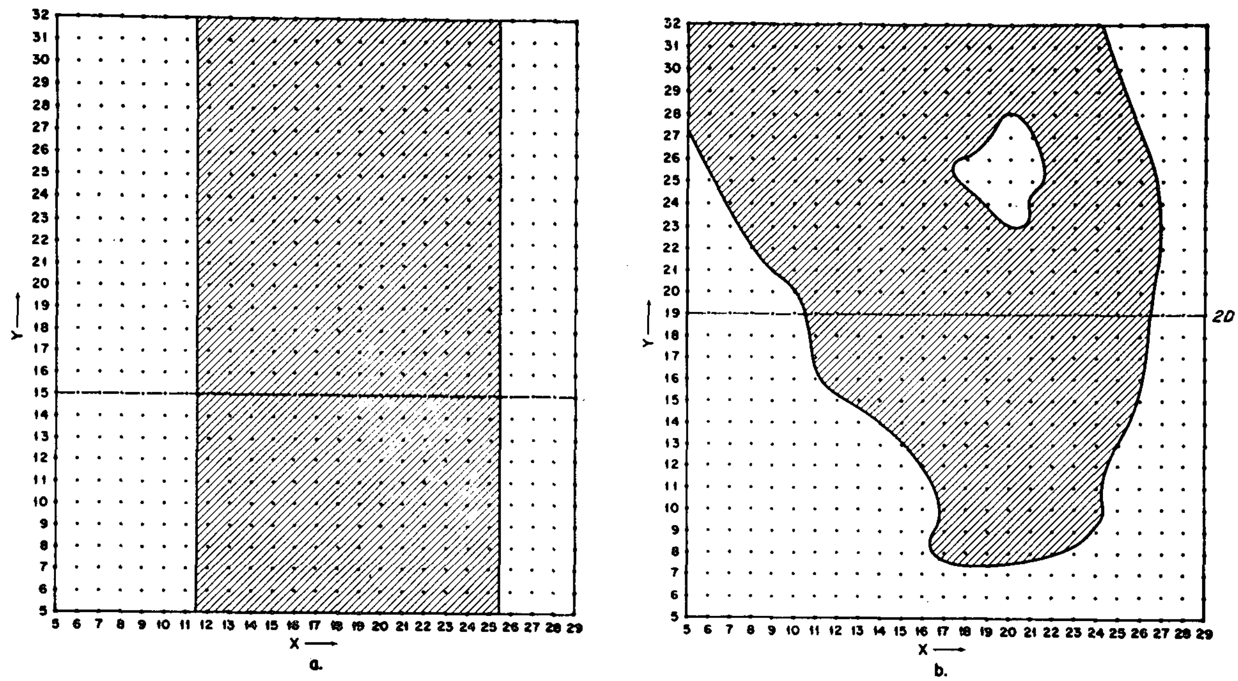


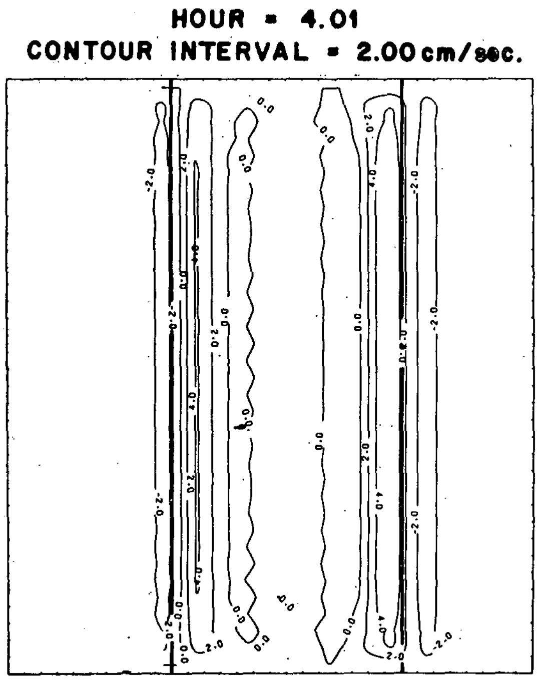
FIG. 1. Model grid for rectangular sea breeze simulation, a., and for South Florida sea breeze simulation, b. The horizontal lines indicate where the two- and three-dimensional models are compared.

line through the figure indicates where the two-dimensional and three-dimensional models are compared.

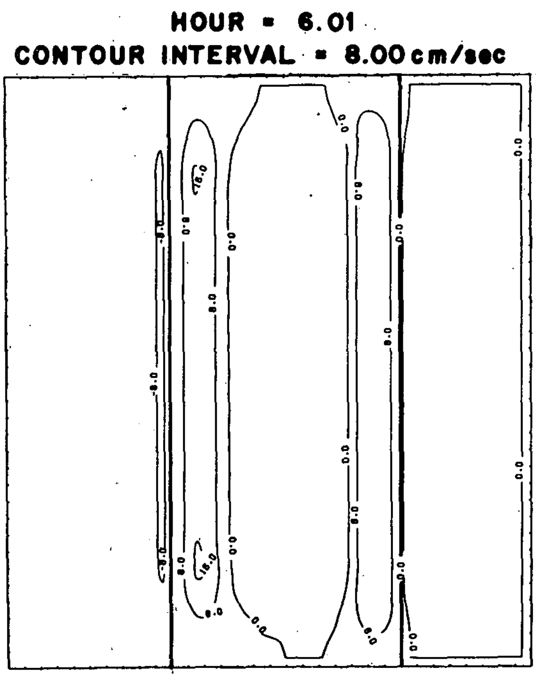
5. Rectangular sea breeze circulations

In this experiment, sea breeze circulations develop over the rectangular shaped land form given in Fig. 1 and interact with an easterly geostrophic wind of 2.5 m sec^{-1}.

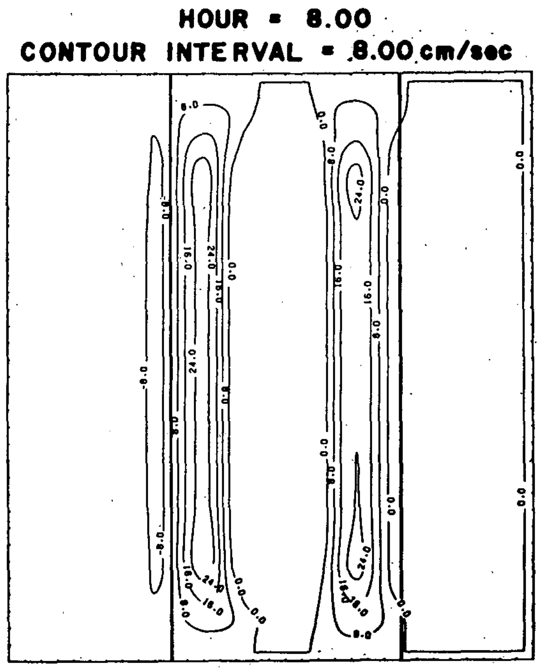
The resultant vertical motion fields at 1.22 km which develop after 4, 6 and 8 hr of integration in the three-dimensional model are illustrated in Fig. 2. The vertical velocities are most intense at this model level. Two regions of upward motion form parallel to both coasts



a.



b.



c.

FIG. 2. The vertical motion field at 1.22 km in the three-dimensional model after 4, 6 and 8 hr for the rectangular sea breeze simulation for a geostrophic wind of 2.5 m sec^{-1} from 90°.

with the eastern (windward) region moving inland slowly during the integration. Except near the lateral boundaries, the vertical motion field is quite homogeneous in the north-south direction.

In order to compare these calculations with the two-dimensional model, data from the three-dimensional model for the vertical plane through the 15th grid point in the y direction after 8 hr of integration are given in Fig. 3a. The comparable results with the same initial conditions and values of prescribed parameters for the two-dimensional model are given in Fig. 3b.

With the value of the arbitrary constant α ($\alpha_{2D}=\alpha_{3D}$) the same in both models, the two-dimensional model predicts substantially higher vertical motion in the two sea breeze convergence zones and substantially stronger subsidence. At first glance, this would not seem to be unexpected since two of the terms in the horizontal exchange coefficient given in (6) are left out in the two-dimensional version given by (7). As seen by the model results in Fig. 2, however, except near the lateral boundaries, the two y-derivative terms must have a negligible influence on the magnitude of the coefficient.

A series of experiments were then performed to see if increasing the value of α_{2D}, which would arbitrarily increase the magnitude of the horizontal exchange coefficient, could decrease the intensity of vertical motion and make the two-dimensional solution more comparable to the three-dimensional result. It was found that a value of α_{2D} about double that in the three-dimensional run gave results, as seen in Fig. 3c, almost identical to those in Fig. 3a for the three-dimensional simulation. Between the two-dimensional model results presented in Fig. 3c and the cross section from the three-dimensional model results presented in Fig. 3a, the solutions differ by no more than 0.1 or 0.2 cm sec^{-1}.

The question immediately arises, then, why the two-dimensional model requires that α be twice as large as in the three-dimensional model. Near the lateral boundaries in the three-dimensional model there are lateral variations, but the region where the two- and three-dimensional calculations are compared appears to be rather homogeneous in the north-south direction.

To see why the two-dimensional model requires additional explicit diffusion, we must examine the generation of vorticity in the three-dimensional model and show that even with two-dimensional forcing, a vertical component of vorticity must form. As explained by Leith (1969) and Lilly (1969), this development of vorticity and its subsequent stretching is the mechanism whereby energy is transferred to smaller scales of motion and finally dissipated on the viscous scale. In a two-dimensional model, by contrast, no vortex stretching is possible so that the energy cannot be transferred properly.

By differentiating (1) with respect to y and (2) with respect to x for the three-dimensional model equa-

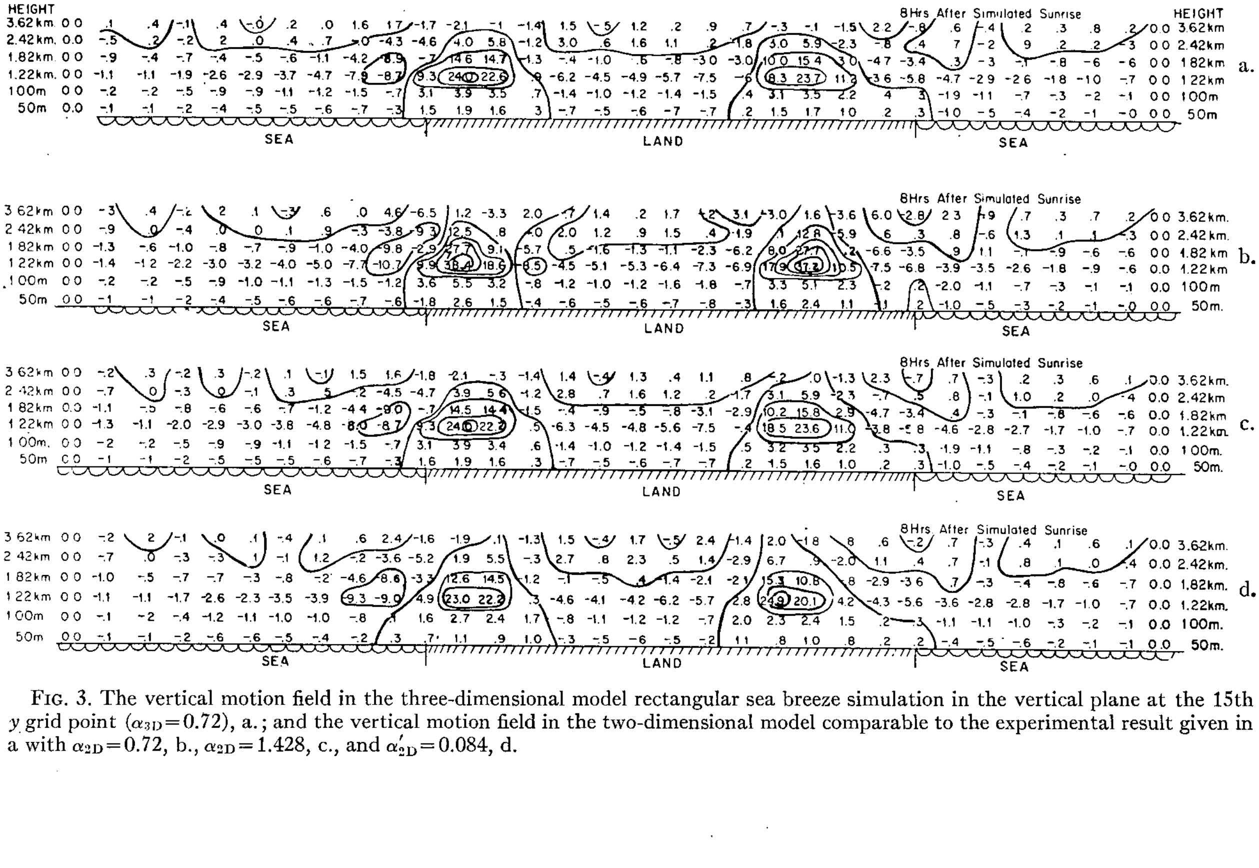


FIG. 3. The vertical motion field in the three-dimensional model rectangular sea breeze simulation in the vertical plane at the 15th y grid point ($\alpha_{3D}=0.72$), a.; and the vertical motion field in the two-dimensional model comparable to the experimental result given in a with $\alpha_{2D}=0.72$, b., $\alpha_{2D}=1.428$, c., and $\alpha'_{2D}=0.084$, d.

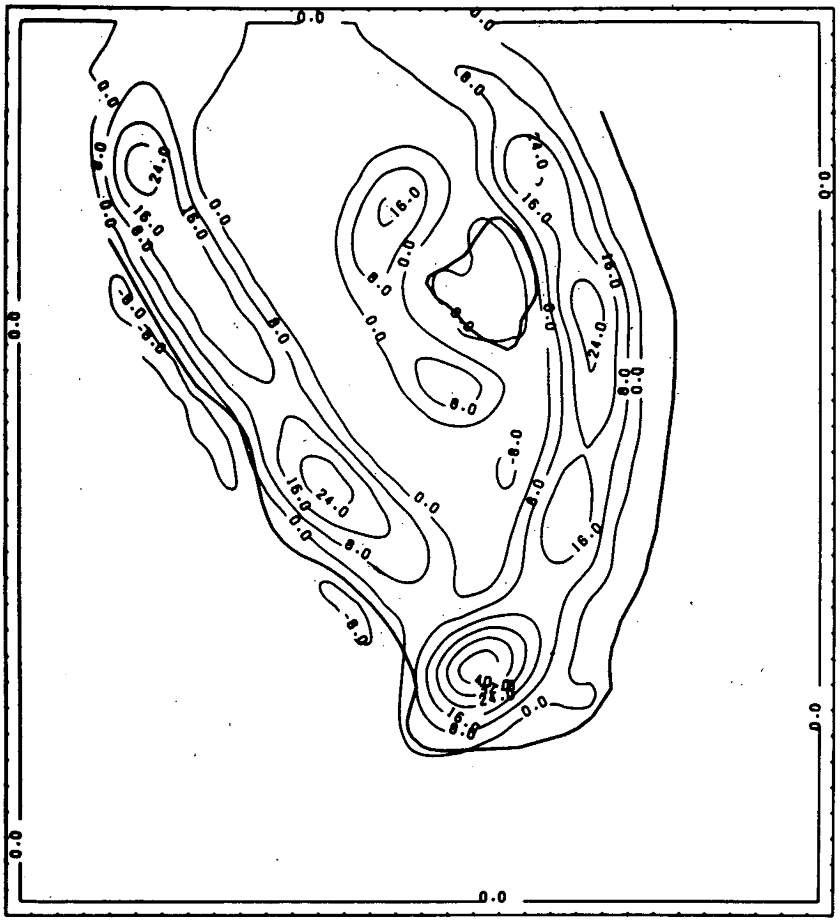


FIG. 4a. The vertical velocity at 1.22 km, 7.5 hr after simulated sunrise on 29 June 1971 for a geostrophic wind of 2.5 m sec⁻¹ from 110° (contour interval 8 cm sec⁻¹).

tions, rearranging the terms, and utilizing the incompressible continuity equation, a vertical vorticity equation of the form

$$\begin{aligned}\frac{d\bar{\xi}_z}{dt}=&\underline{\frac{\partial\bar{w}}{\partial z}(f+\bar{\xi}_z)}-\underline{\frac{\partial\bar{w}}{\partial x}\frac{\partial}{\partial z}}(\bar{v}+\hat{v})+\frac{\partial\bar{w}}{\partial y}\frac{\partial}{\partial z}(\bar{u}+\hat{u})\\&+\underline{\frac{f}{\bar\theta}\left(u_g\frac{\partial\bar\theta}{\partial x}+v_g\frac{\partial\bar\theta}{\partial y}\right)}+\hat{f}\frac{\partial\bar w}{\partial y}+K_H^{(m)}\frac{\partial^2\bar\xi}{\partial x^2}\\&+K_H^{(m)}\frac{\partial^2\bar\xi}{\partial y^2}+[\hat K_z^{(m)}+\tilde K_z^{(m)}]\frac{\partial^2\bar\xi}{\partial z^2}+\bar\xi\frac{\partial^2}{\partial y\partial x}K_H^{(m)}\\&+\frac{\partial}{\partial z}(\bar v-\bar u)\left\{\frac{\partial^2}{\partial x\partial y}[\hat K_z^{(m)}+\tilde K_z^{(m)}]\right\}\\&+\frac{\partial}{\partial z}(\hat v-\hat u)\left[\frac{\partial^2}{\partial x\partial y}\tilde K_z^{(m)}\right]\\&+\underline{\frac{\partial\bar v}{\partial x}\frac{\partial^2}{\partial x^2}}K_H^{(m)}-\frac{\partial\bar u}{\partial y}\frac{\partial^2}{\partial y^2}K_H^{(m)}\qquad(9)\end{aligned}$$

is produced, where

$$\bar\xi_z=\frac{\partial\bar v}{\partial y}-\frac{\partial\bar u}{\partial x}.$$

The first term on the right of (9) is the product of the absolute vertical vorticity and the horizontal divergence, while the next two terms are the so-called tilting terms. The next term is the solenoid term, while the remaining terms represent a contribution to vorticity change as a result of the w-Coriolis term from the u equation of motion and as a result of horizontal and vertical diffusion effects.

It is evident from (9), as denoted by the underlined terms, that vertical vorticity can be generated as a result of variations in the x-z plane with no y variations required. What this means, of course, is that even with two-dimensional forcing, the physical response of the system is to develop three-dimensional circulations which must eventually be dissipated by the parameterized diffusion. Since three-dimensional circulations cannot develop in the two-dimensional model, the horizontal diffusion must be proportion-

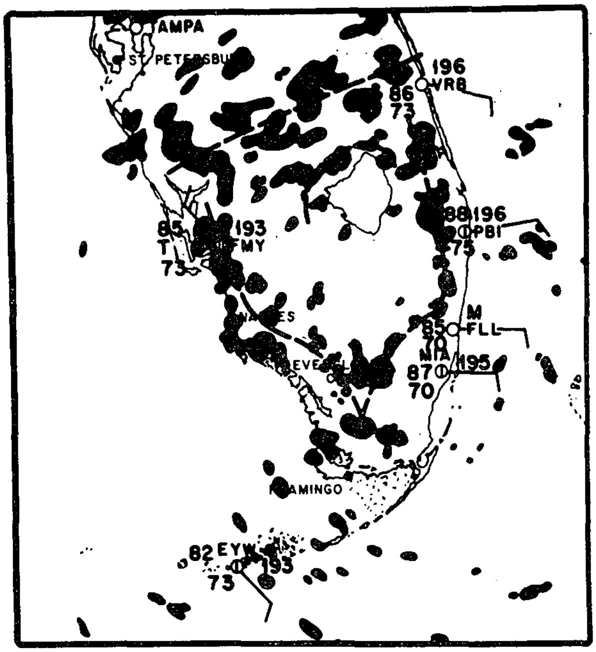


FIG. 4b. The radar depiction from the Miami WSR-57 radar, 7 hr 28 min after sunrise on 29 June 1971.

ately increased in order to parameterize the proper transfer of energy even in the resolvable scales. This reinforces Deardorff's (1973) contention that even with strong two-dimensional forcing such as in two-dimensional sea breeze models, the circulations which develop are so inherently three dimensional that the Reynold-stress terms must represent the resolvable momentum fluxes as well as the subgrid-scale fluxes.

This experiment, then, suggests that two-dimensional sea breeze models can produce identical solutions to a three-dimensional sea breeze model, with two-dimensional forcing, but only when the horizontal flux terms in the u equation of motion and the thermodynamic equation are adjusted so as to represent a portion of the resolvable fluxes as well as the subgrid-scale processes.

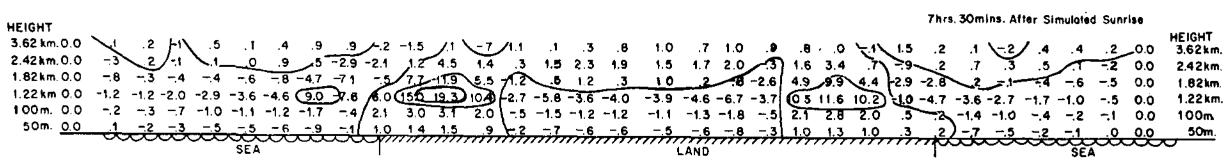
The final experiment performed in this section used the two-dimensional model with the horizontal exchange coefficient as given by (8). It was found that an α_{2D}' value of about 0.084 gave the closest agreement with the three-dimensional simulation, although the maximum magnitude in the convergence zones was somewhat displaced. The vertical motion field 8 hr after simulated sunrise is given in Fig. 3d.

It is somewhat surprising that the vorticity form of the horizontal exchange coefficient produces poorer solutions than the deformation form of the horizontal exchange coefficient since, as suggested by Leith (1969) and Lilly (1969), a vorticity form of the coefficient is appropriate for two-dimensional simulations. However, both Leith and Lilly were discussing turbulence simulations in which the grid scale lies within an inertial sub-range. In the models discussed here, this certainly is not true; the grid separation lies in the production portion of the energy spectrum. There is, therefore, no guarantee that the vorticity form is the most appropriate form and the results suggest that it is not.

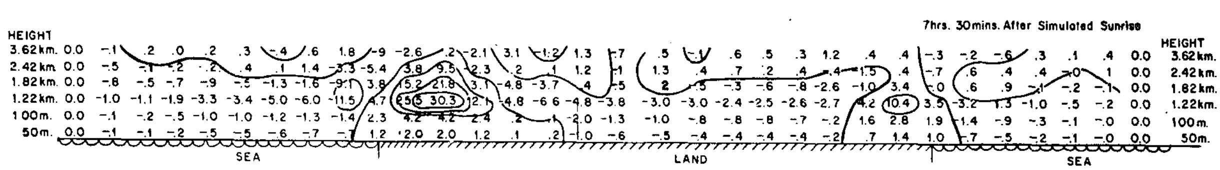
6. South Florida sea breeze circulation

Now we must examine the more relevant question: Can a two-dimensional model, with suitably adjusted diffusion, provide accurate solutions to actual three-dimensional sea breeze situations in the real atmosphere? To do this, results from a three-dimensional model simulation for 29 June 1971 over south Florida [discussed by Pielke (1974)] are compared against a two-dimensional model run which used the same initial conditions and values of prescribed parameters except, based on the results of the last section, that α_{2D} was made about double that of α_{3D}. In this experiment the geostrophic wind was 2.5 m sec^{-1} from 110°.

The vertical motion field at 1.22 km after 7.5 hr as predicted by the three-dimensional model, along with the shower locations obtained from the Miami WSR-57 10-cm radar at the equivalent time, are given in Fig. 4. As shown by Pielke (1974), the locations of the



a.



b.

FIG. 5. The vertical motion field in the three-dimensional model 29 June 1971 simulation in the vertical plane at the 19th y-grid point ($\alpha_{3D}=0.72$), a., and in the two-dimensional model simulation with ($\alpha_{2D}=1.428$), b.

showers are well correlated with the regions of most intensive vertical motion.

In order to compare the two models, a vertical cross section through the 19th y-grid point in the three-dimensional model is compared against the two-dimensional model results which used the same specification of the land-sea interface as in the three-dimensional cross section. The cross section, shown in Fig. 5a, was chosen as that region over south Florida where the two dimensionality of the coastline would most likely be satisfied. The equivalent two-dimensional calculation is given in Fig. 5b.

As is evident in the figure, the two solutions are markedly different. Along the leeward coastline the maximum vertical velocities are over the same grid point in both models, but the values are substantially larger in the two-dimensional simulation. Along the windward coast in the three-dimensional model the zone of maximum upward motion is two grid points (22 km) further inland and has slightly larger magnitudes than in the two-dimensional case. It is clear from these results, in contrast to the rectangular sea breeze simulation, that the solutions obtained from the two- and three-dimensional models cannot be forced to agree by adjusting the value of α_{2D}. Along the lee coast the vertical velocities are much larger in the two-dimensional model, while along the windward coasts the magnitudes are similar. It therefore appears that a two-dimensional simulation of the sea breezes over south Florida is inadequate.

It may be argued, however, that a two-dimensional simulation can permit higher vertical resolution, since lateral grid points are not needed, and can thereby improve the reality of the solutions relative to a lower resolution three-dimensional simulation.

To check this possibility, calculations were performed with the two-dimensional model for the case discussed in Section 5 with $\alpha_{2D}=1.428$, where the number of grid points was increased to 14. Since the two- and three-dimensional models agree for this case, any change in the solution due to the increased resolution should indicate an improvement in the accuracy. In the first case, the resolution was added on the top of the model at 1.2-km intervals, extending the depth of the model to about 12 km. As seen in Fig. 6a, the solution after 7.5 hr was almost identical to that in Fig. 3c. This confirms that the sea breeze case discussed here is basically confined to the lower levels of the atmosphere and the added resolution at the upper levels adds little to the dry sea breeze simulation.

In the second run with added vertical resolution, grid points were added below the original top of the model midway between the original grid points. As seen in Fig. 6b, although the locations of the maximum vertical velocity are over the same areas, the magnitudes are somewhat different than in Fig. 3c, particularly in the eastern convergence zone where the

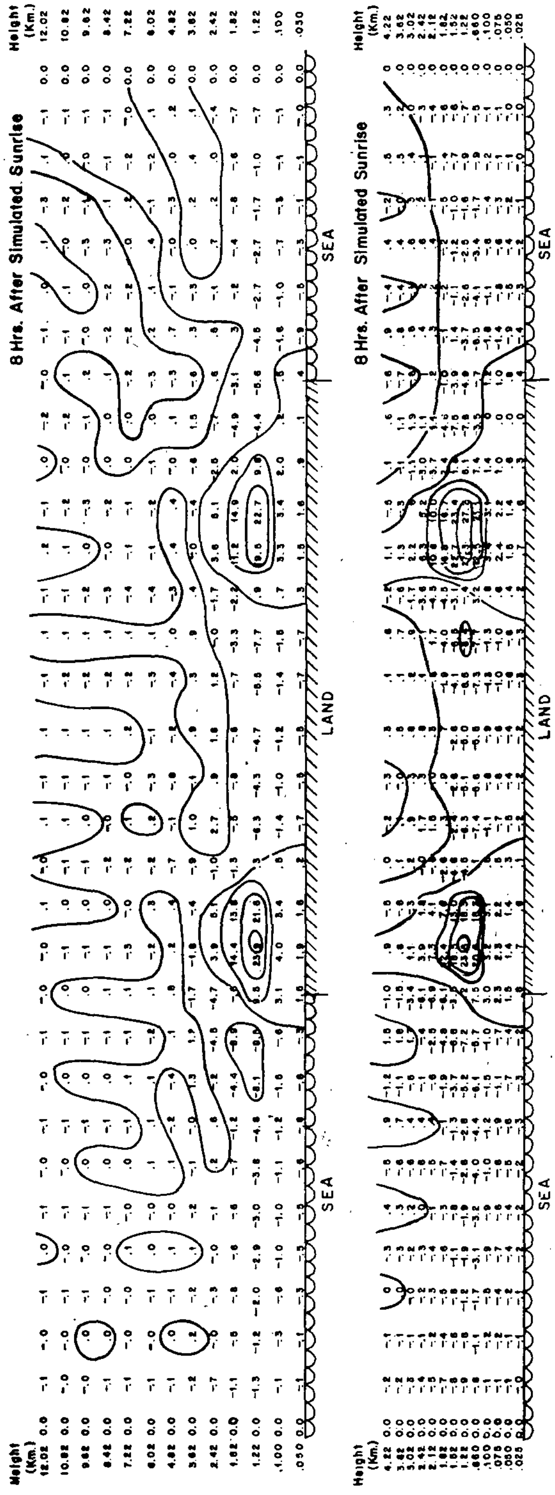


FIG. 6. The vertical motion field in the two-dimensional model with ($\alpha_{2D}=1.428$) for the rectangular sea breeze simulation, equivalent to the results presented in FIG. 3c except for seven additional levels placed above, a., and below, b., the initial level of the material surface given in that experiment.

vertical motion is somewhat larger. Nevertheless, there is less difference between Figs. 3c and 6b than between Figs. 5a and 5b, which implies that even though the added vertical resolution produces more detailed vertical structure, an inclusion of the lateral geographic variations over south Florida is required to obtain realistic sea breeze simulations.

7. Conclusions

The experiments performed for this paper have yielded two major conclusions. First, the results demonstrate that a two-dimensional numerical model of the sea breeze can produce identical solutions to a three-dimensional numerical model of the sea breeze, but only when the forcing is two dimensional, and only when the explicit horizontal diffusion is increased to include resolvable as well as subgrid-scale horizontal fluxes. Second, the results from the second experiment demonstrate the inability of the two-dimensional model to provide an accurate simulation of the sea breeze even over a region in south Florida where the forcing appears most two dimensional and even with added vertical resolution. The conclusion must be made that although a two-dimensional sea breeze model is a valuable tool with which to examine the theoretical properties of the dry sea breeze generation and evolution, a full three-dimensional sea breeze model is required to obtain realistic solutions to most practical problems.

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APPENDIX

List of symbols

- $\tilde{u}$ grid-volume averaged east-west velocity perturbation
- $\tilde{v}$ grid-volume averaged north-south velocity perturbation
- $\tilde{\theta}$ grid-volume averaged potential temperature perturbation
- $\tilde{\Pi}$ grid-volume averaged scaled pressure perturbation $[=\hat{\Pi}+\tilde{\Pi}=c_p(p/p_{00})^{R/c_p}]$
- c_p specific heat at constant pressure
- p total resolvable pressure
- p_{00} reference pressure
- R gas constant for dry air
- $\tilde{w}$ grid-volume averaged vertical velocity perturbation
- $\hat{u}$ synoptic-scale east-west velocity
- $\hat{v}$ synoptic-scale north-south velocity
- $\hat{\theta}$ synoptic-scale potential temperature
- $\hat{\Pi}$ synoptic-scale scaled pressure
- $\hat{w}$ synoptic-scale vertical velocity
- x east-west spatial coordinate
- y north-south spatial coordinate
- z vertical spatial coordinate
- t time coordinate
- f Coriolis parameter $[=2\Omega \sin\phi]$
- $\hat{f}$ $[=2\Omega \cos\phi]$
- Ω angular velocity of the earth
- ϕ latitude
- $\hat{K}_z^{(m)}$ synoptic-scale vertical eddy exchange coefficient for momentum
- $\hat{K}_z^{(\theta)}$ synoptic-scale vertical eddy exchange coefficient for heat
- $\tilde{K}_z^{(m)}$ perturbation from $\hat{K}_z^{(m)}$
- $\tilde{K}_z^{(\theta)}$ perturbation from $\hat{K}_z^{(\theta)}$
- K_H horizontal eddy exchange coefficient (identical for heat and momentum)
- α_{3D} adjustable coefficient in the three-dimensional version of K_H
- $\left.\begin{matrix}\alpha_{2D}\\ \alpha'_{2D}\end{matrix}\right\}$ adjustable coefficients in the two-dimensional versions of K_H
- g gravity
- u_g geostrophic east-west velocity
- v_g geostrophic north-south velocity

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