NOTES AND CORRESPONDENCE

Vertical Normal Modes of a Mesoscale Model Using a Scaled Height Coordinate

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ABSTRACT

Vertical modes were derived for a version of the Colorado State Regional Atmospheric Mesoscale Modeling System. We studied the impacts of three options for dealing with the upper boundary of the model. The standard model formulation holds pressure constant at a fixed altitude near the model top, and produces a fastest mode with a speed of about 90 m s^{-1}. An alternative formulation, which allows for an external mode, could require recomputation of vertical modes for every surface elevation on the horizontal grid unless the modes are derived in a particular way. These results have bearing on the feasibility of applying vertical mode initialization to models with scaled height coordinates.

1. Introduction

Vertical mode initialization (VMI) has been applied productively to regional scale models (Bourke and McGregor, 1983) and has the potential of being applied to mesoscale models. Such an application may be useful when high-resolution sounding data is used to initialize the mesoscale model domain. This could be done when special radiosonde soundings are available, or when satellite-based radiometers, such as VAS (Menzel et al., 1983), provide high resolution soundings.

Bourke and McGregor (1983; hereafter denoted BM) demonstrated an initialization technique which relied on formulating the model dynamical equations such that terms in the linear shallow water equations,

$$\dot{D} = -\nabla^2\phi + f\zeta \tag{1a}$$

$$\dot{\zeta} = -fD \tag{1b}$$

$$\dot{\phi} = -gHD \tag{1c}$$

were isolated. In Eqs. (1) D is divergence, ϕ is geopotential height, ζ is vorticity, f is the Coriolis parameter, H is the height of the fluid, and $(\dot{\ }) = \partial(\ \ )/\partial t$. The full model equations were thus written for the BM approach as

$$\dot{D} = -\nabla^2\phi + f\zeta + N_D \tag{2a}$$

$$\dot{\zeta} = -fD + N_\zeta \tag{2b}$$

$$\dot{\phi} = -gHD + N_\phi, \tag{2c}$$

where the N represent nonlinear and residual terms. Bourke and McGregor demonstrated how gravity waves could be eliminated from (2) by setting $\dot{D} = \dot{\phi} = 0$ or by setting $\dot{D} = f\dot{\xi} - \nabla^2\dot{\phi} = 0$.

In this study the equations of one version of the Colorado State Regional Atmospheric Mesoscale Modeling System (RAMMS) were manipulated into the form of (2), and vertical modes were derived. This was done in anticipation of initializing the RAMMS with nonhomogeneous fields of wind and temperature data from a mesoscale sounding system.

2. Reformulation of the RAMMS equations

The version of RAMMS used in this study is hydrostatic and incompressible. The basic formulation is summarized in Mahrer and Pielke (1977), and in Pielke (1984; pp. 111-126). A salient feature of the model is its vertical coordinate, which is a scaled height that accounts for variable terrain height, z_G. Unlike the Mahrer and Pielke (1977) version, scaling is done to a fixed height $\bar{s}$, such that

$$z^* = \bar{s}\frac{z - z_G}{\bar{s} - z_G}.$$

The height $\bar{s}$ is the initial height of a material surface s, which serves as the top of the model atmosphere. While $\bar{s}$ is a constant, s varies horizontally and with time. The vertical coordinate and material surface will be shown to have a substantial impact on derivation of normal modes.

A 26-level version of the model was used (25 regular levels plus the material surface), with values of z^* ranging from 10 to 13 500 m. With the above definition of z^*, the model equations may be abbreviated as

$$\frac{\partial u}{\partial t} = fv - \theta_0\frac{\partial \pi}{\partial x} + N_u \tag{3}$$

$$\frac{\partial v}{\partial t} = -fu - \theta_0\frac{\partial \pi}{\partial y} + N_v \tag{4}$$

$$\frac{d\theta}{dt} = N_\theta \tag{5}$$

$$(\bar{s}-z_G)\frac{\partial w^*}{\partial z^*}+\frac{\partial}{\partial x}[u(\bar{s}-z_G)]+\frac{\partial}{\partial y}[v(\bar{s}-z_G)]=0 \qquad (6)$$

$$\frac{\partial \pi}{\partial z^*}=-\frac{\bar{s}-z_G}{\bar{s}}\frac{g}{\theta}\equiv -S\frac{g}{\theta}, \qquad (7)$$

where u, v and w^* are the velocity components, π is the Exner function, θ is potential temperature, θ_0 is the horizontally domain-averaged potential temperature, and the symbol N represents the nonlinear terms.

In working (3)–(7) toward the form of (2) an expression is needed for the horizontal divergence D. This can be easily inferred from the continuity equation (6), which can be written as

$$\frac{\partial w^*}{\partial z^*}=-\frac{1}{(\bar{s}-z_G)}\left\{\frac{\partial}{\partial x}[u(\bar{s}-z_G)]+\frac{\partial}{\partial y}[v(\bar{s}-z_G)]\right\}, \qquad (8)$$

or

$$\frac{\partial w^*}{\partial z^*}=-D. \qquad (9)$$

Equation (9) can be integrated from the ground surface ($w_b^*=0$) up to an arbitrary level z^* to yield a diagnostic equation for vertical velocity,

$$w^*=-\int_0^{z^*} D dz^*. \qquad (10)$$

a. Divergence tendency equation

An equation of the form (2a) can be derived from the horizontal momentum equations (3) and (4) by first multiplying through by $(\bar{s}-z_G)$, giving

$$\frac{\partial}{\partial t}[u(\bar{s}-z_G)] = f[v(\bar{s}-z_G)]-\theta_0(\bar{s}-z_G)\frac{\partial \pi}{\partial x}+N_u(\bar{s}-z_G), \qquad (11)$$

$$\frac{\partial}{\partial t}[v(\bar{s}-z_G)] = -f[u(\bar{s}-z_G)]-\theta_0(\bar{s}-z_G)\frac{\partial \pi}{\partial y}+N_v(\bar{s}-z_G). \qquad (12)$$

An equation for $\dot{D}$ results from taking the derivative of (11) with respect to x, and subtracting the derivative of (12) with respect to y, and then dividing the result by $(\bar{s}-z_G)$. That process results in

$$\dot{D}=f\zeta-\theta_0\nabla^2\pi+N_D, \qquad (13)$$

where several terms have been combined into N_D, ∇^2 is the horizontal Laplacian, and

$$\zeta=\frac{1}{(\bar{s}-z_G)}\left\{\frac{\partial}{\partial x}[v(\bar{s}-z_G)]-\frac{\partial}{\partial y}[u(\bar{s}-z_G)]\right\}. \qquad (14)$$

For convenience θ_0 can be included inside the Laplacian since it does not vary horizontally. Thus

$$\dot{D}=f\zeta-\nabla^2(\theta_0\pi)+N_D. \qquad (15)$$

b. Vorticity tendency equation

The equation for $\dot{\zeta}$ is solved for in a manner very similar to (15), starting with (11) and (12). In this case the x-derivative is applied to (12), and the y-derivative of (11) is subtracted from it. The result is

$$\dot{\zeta}=-fD+N_\zeta. \qquad (16)$$

c. Pressure tendency equation

The discretized pressure tendency equation is expected to take the form (BM)

$$(\theta_0\dot{\pi})_k=-\sum_{n=1}^{K} C_{kn}D_n+N_{\pi k},$$

where k represents a model level and K is the number of levels. In general C includes two parts. First, there is a term for changes in pressure at a boundary z_b^*. The boundary is usually taken as the ground surface ($z_b^*=0$), but another option is addressed below. Second, there is a term to account for divergence between the boundary and k. The form of the second term does not depend on the choice of z_b^*, so it will be concentrated on first.

The pressure (Exner function) at some given level z^* can be found by integrating the quantity $\partial\pi/\partial z^*$ from z_b^* to z^*. Taking the local time derivative yields

$$\dot{\pi}(z^*)-\dot{\pi}(z_b^*)=\int_{z_b^*}^{z^*}\frac{\partial}{\partial t}\left(\frac{\partial \pi}{\partial z^*}\right)dz^*, \qquad (17)$$

where z^* and z_b^* must be independent of time to allow for bringing the time derivative inside the integral. An expression for $(\partial^2\pi/\partial t dz^*)$ in terms of D must be found so that (17) can ultimately take the form of (2c). A rewarding approach is to start with the hydrostatic equation (7), rewritten as

$$\frac{1}{\theta}=-\frac{1}{gS}\frac{\partial \pi}{\partial z^*}. \qquad (18)$$

Taking the total derivative yields

$$\frac{d}{dt}\left(\frac{1}{\theta}\right)=-\frac{1}{\theta^2}\frac{d\theta}{dt}=-\frac{1}{gS}\frac{d}{dt}\left(\frac{\partial \pi}{\partial z^*}\right)-\frac{\partial \pi}{\partial z^*}\frac{1}{g}\frac{d}{dt}\left(\frac{1}{S}\right). \qquad (19)$$

Rearranging (19) we can solve for $d\theta/dt$, such that

$$\frac{d\theta}{dt}=\frac{\theta^2}{gS}\frac{d}{dt}\left(\frac{\partial \pi}{\partial z^*}\right)+\frac{\partial \pi}{\partial z^*}\frac{\theta^2}{g}\frac{d}{dt}\left(\frac{1}{S}\right)=N_\theta, \qquad (20)$$

where the thermodynamic equation (5) has been invoked. Further manipulation yields

$$\frac{d}{dt}\left(\frac{\partial\pi}{\partial z^*}\right) = N'_\theta, \tag{21}$$

where the third and fourth terms of (20) have been included in N'_θ. Expanding the total derivative and rearranging gives

$$\frac{\partial}{\partial t}\left(\frac{\partial\pi}{\partial z^*}\right) = -w^* \frac{\partial}{\partial z^*}\left(\frac{\partial\pi}{\partial z^*}\right) - \mathbf{V}\cdot\nabla\left(\frac{\partial\pi}{\partial z^*}\right) + N'_\theta. \tag{22}$$

A more convenient expression for $\partial(\partial\pi/\partial z^*)/\partial z^*$ can be derived through taking the vertical derivative of the hydrostatic equation and separating it into its domain averaged (θ_0) and perturbation (θ') components, so that

$$\frac{\partial}{\partial z^*}\left(\frac{\partial\pi}{\partial z^*}\right) = Sg\frac{1}{(\theta_0+\theta')^2}\frac{\partial}{\partial z^*}(\theta_0+\theta') \tag{23}$$

$$\approx S\frac{g}{\theta_0^2}\frac{\partial\theta_0}{\partial z^*} + S\frac{g}{\theta_0^2}\frac{\partial\theta'}{\partial z^*}, \tag{24}$$

so

$$\frac{\partial}{\partial z^*}\left(\frac{\partial\pi}{\partial z^*}\right) \approx \gamma_0 + \gamma'. \tag{25}$$

Here γ_0 and γ' are the mean and perturbation static stabilities, respectively. Substituting (25) back into (22) yields

$$\frac{\partial}{\partial t}\left(\frac{\partial\pi}{\partial z^*}\right) = -w^*\gamma_0 - w^*\gamma' - \mathbf{V}\cdot\nabla\left(\frac{\partial\pi}{\partial z^*}\right) + N'_\theta \tag{26}$$

$$= -w^*\gamma_0 + N_\gamma, \tag{27}$$

where the rightmost terms have been included in N_γ.

Equation 10 can be substituted back into (27), and the result then substituted into (17) to arrive at

$$\dot\pi(z^*) - \dot\pi(z_b^*) = \int_{z_b^*}^{z^*}\left[\gamma_0\int^{z^*} D\,dz^* + N_\gamma\right]dz^*. \tag{28}$$

To continue the derivation of the pressure tendency equation we must deal with the boundary term. Three types of boundary terms were considered. Case 1 is the standard condition of the RAMMS, in which $z_b^* = \bar{s}^*$ ($=\bar{s}$) and $\pi(\bar{s}^*)$ is constant in time. Such a condition prevents gravity waves from propagating through level $\bar{s}^*$. Therefore, there can be no external gravity mode in case 1, although fast internal modes are still possible. Cases 2 and 3 were both based on the condition that $\pi(s^*)$ is constant. They allow external waves and provide for comparison with results of other studies. The second and third cases differ in that for case 2, $z_b^* = 0$, and for case 3, $z_b^* = \bar{s}^*$.

Case 1:

$$z_b^* = \bar{s}^*, \quad \dot\pi(\bar{s}^*) = 0.$$

Since $\dot\pi(z_b^*) = 0$ for this case, there is no boundary term. Also, the first limit of the outside integration in (28) is at the top of the model.

In discrete form (28) becomes

$$\dot\pi_k = -\sum_{j=k}^{M}[\gamma_{0j}\sum_{n=1}^{j} D_n\delta_{nj} + N_{\gamma j}]d_{jk} - \gamma_{0M}D_M\Delta z_M^* d_{Mk},$$
$$(1 \leq k \leq M-1), \tag{29}$$

where M denotes the model level corresponding to $\bar{s}^*$. The last term accounts for the divergence on $\bar{s}^*$, which is constant throughout the model integration. In (29) the vertical differencing factors are given by

$$\delta_{nj} = \begin{cases}\Delta z_n^*, & n \leq j, \quad j \neq M\\ 0, & n > j \quad \text{or} \quad j = M\end{cases}$$

for integration to level j from $z^* = 0$, and

$$d_{jk} = \begin{cases}0, & j<k\\ \Delta z_j^*, & j\geq k\end{cases}$$

for integration from level k to $\bar{s}^*$. The values of Δz_i^* are vertical grid lengths centered on grid points i, except at boundaries of integration, where they are half grid lengths.

The nonlinear terms can be separated out as

$$\dot\pi_k = -\sum_{j=k}^{M} N_{\gamma j}d_{jk} - \gamma_{0M}D_M\Delta z_M^* d_{Mk} - \sum_{j=k}^{M}\gamma_{0j}[\sum_{n=1}^{j} D_n\delta_{nj}d_{jk}]. \tag{30}$$

This equation is still not satisfactory in this form, since the goal was to arrive at an equation with D_n as an isolated multiplicative factor. This condition can be remedied by noting that the definition of δ allows the summation over n to be carried from 1 to K ($K \geq$ all j) without altering the value of the interior summation. For the quadrature used here $\delta_{nj} = 0$ for $n \geq M$, so K can be taken as $M - 1$, which is the highest computational level of the model. With the new limits on summation (not a function of j), the interior summation can be moved to the outside, resulting in

$$\dot\pi_k = R_{\pi k}^1 - \sum_{n=1}^{K}[\sum_{j=k}^{M}\gamma_{0j}\delta_{nj}d_{jk}]D_n. \tag{31}$$

Here R is used as shorthand for the first two terms of (30), with the superscript identifying case 1. For consistency with (15) this should be multiplied through by θ_{0k}, yielding

$$(\dot{\theta_0\pi})_k = \theta_{0k}R_{\pi k}^1 - \sum_{n=1}^{K}[\theta_{0k}\sum_{j=k}^{M}\gamma_{0j}\delta_{nj}d_{jk}]D_n, \tag{32}$$

or

$$(\dot{\theta_0\pi})_k = N_{\pi k}^1 - \sum_{n=1}^{K} C_{kn}^1 D_n, \tag{33}$$

which is the appropriate pressure tendency equation for case 1.

Case 2:

$$z_B^* = 0, \quad \dot{\pi}(s^*) = 0.$$

The surface pressure tendency, $\dot{\pi}(0)$, can be derived by vertically integrating the hydrostatic equation over the depth of the model atmosphere, and appropriately defining a π-weighted average potential temperature $\bar{\theta}$; yielding

$$\pi(0) = \pi(s^*) + \frac{Sgs^*}{\bar{\theta}}. \tag{34}$$

Differentiation with respect to time and dropping terms equal to zero results in

$$\dot{\pi}(0) = \frac{Sg}{\bar{\theta}}\frac{\partial s^*}{\partial t} - \frac{Sgs^*}{\bar{\theta}^2}\frac{\partial \bar{\theta}}{\partial t}. \tag{35}$$

An expression for $\partial s^*/\partial t$ is found by noting that

$$\frac{ds^*}{dt} = w_s^* = \frac{\partial s^*}{\partial t} + \mathbf{V}\cdot\nabla s^*. \tag{36}$$

Substituting (36) into (35) results in

$$\dot{\pi}(0) = \frac{Sg}{\bar{\theta}} w_s^* + R_2, \tag{37}$$

where two terms have been included in R_2. Application of (10) yields

$$\dot{\pi}(0) = -\frac{Sg}{\bar{\theta}}\int_0^{z^*} Ddz^* + R_2. \tag{38}$$

Discretization of (28) can be carried out as for case 1, but using (38) as the boundary term, and performing all summations relative to the lower boundary. This results in

$$(\dot{\theta_0\pi})_k = N_{\pi k}^2 - \sum_{n=1}^{K} C_{kn}^2 D_n, \tag{39}$$

where, as before, several nonlinear terms have been included in $N_{\pi k}^2$, and

$$C_{kn}^2 = \theta_{0k}\left[\frac{Sg}{\bar{\theta}}\delta_{nK} - \sum_{j=1}^{k}\gamma_{0j}\delta_{nj}\delta_{jk}\right]. \tag{40}$$

Case 3:

$$z_B^* = \bar{s}^*, \quad \dot{\pi}(s^*) = 0.$$

The third case is essentially a hybrid of cases 1 and 2. Here $\dot{\pi}(z^*)$ is computed relative to the condition on π at $\bar{s}^*$, but $\dot{\pi}(\bar{s}^*) \neq 0$.

The equation for $\dot{\pi}(\bar{s}^*)$ is similar to (35), with the alteration that the average on θ is carried out over the interval $\bar{s}^*$ to s^*, such that

$$\dot{\pi}(\bar{s}^*) = \frac{Sg}{\hat{\theta}}\frac{\partial s^*}{\partial t} - \frac{Sg}{\hat{\theta}^2}(s^* - \bar{s}^*)\frac{\partial \hat{\theta}}{\partial t}, \tag{41}$$

where $\hat{\theta}$ is the new averaged θ. Equation 41 has an advantage over (35) in that at initialization time $s^* = \bar{s}^*$, so $\hat{\theta} = \theta(\bar{s}^*)$, and

$$\dot{\pi}(\bar{s}^*) = \frac{Sg}{\theta(\bar{s}^*)}\frac{\partial s^*}{\partial t}. \tag{42}$$

Use of (42) as the boundary term results in a discretized pressure tendency equation

$$(\dot{\theta_0\pi})_k = N_{\pi k}^3 - \sum_{n=1}^{K} C_{kn}^3 D_n, \tag{43}$$

with

$$C_{kn}^3 = \theta_{0k}\left[\frac{Sg}{\theta(\bar{s}^*)}\delta_{nk} + \sum_{j=k}^{M}\gamma_{0j}\delta_{nj}d_{jk}\right]. \tag{44}$$

The equations (15), (16) and either (33), (39) or (43) are a complete set with shallow water terms isolated, and are coupled via the vertical structure matrices $\mathbf{C}^1$, $\mathbf{C}^2$ or $\mathbf{C}^3$. Decoupling is accomplished by first solving the eigenvalue problem

$$\sum_{n=1}^{K} C_{kn}E_{nj} = E_{kj}\lambda_j \tag{45}$$

for the eigenvalues λ_j, and for the eigenvectors composing the matrix whose elements are E_{ij}, as was done by BM. The quantities D, ζ and $(\theta_0\pi)$ can be expanded in terms of their vertical modes $\mathcal{D}$, ξ and $\mathcal{P}$,

$$D_k = \sum_{j=1}^{K} E_{kj}\mathcal{D}_j, \quad \zeta_k = \sum_{j=1}^{K} E_{kj}\xi_j, \quad (\theta_0\pi)_k = \sum_{j=1}^{K} E_{kj}\mathcal{P}_j, \tag{46}$$

and the expansions substituted into (15), (16) and (33), (39) or (43). As in BM, left multiplication of the resultant equations by $(\mathbf{E}^{-1})_{ij}$, with summation over all k produces the decoupled equations

$$\dot{\mathcal{D}}_i = N_{\mathcal{D}i} + f\xi_i - \nabla^2\mathcal{P}_i \tag{47}$$

$$\dot{\xi}_i = N_{\xi i} - f\mathcal{D}_i \tag{48}$$

$$\dot{\mathcal{P}}_i = N_{\mathcal{P}i} - \lambda_i\mathcal{D}_i. \tag{49}$$

3. Vertical modes

An important result of the formulation of (33), (39) and (43) is that the vertical structure matrices $\mathbf{C}$ are dependent on the ground height z_G via the boundary term and the static stability term γ_0, which includes S $[S = (\bar{s} - z_G)/\bar{s}]$. That dependence implies that $\mathbf{C}$ varies in horizontal space within the model domain. The spatial dependence manifested in the term S is just a multiplicative factor in all terms and elements of $\mathbf{C}$. The term S can be factored out such that

$$S\sum_{n=1}^{K} C'_{kn}E_{nj} = E_{kj}\lambda_j, \tag{50}$$

or

$$\sum_{n=1}^{K} C'_{kn}E_{nj} = E_{kj}\lambda'_j, \tag{51}$$

TABLE 1. Heights and potential temperatures used in computing vertical modes.

Level	z^* (m)	θ_0 (K)	Level	z^* (m)	θ_0 (K)
1	10	303.9	14	2 750	317.5
2	20	304.0	15	3 250	318.8
3	30	304.1	16	3 750	320.0
4	50	304.2	17	4 250	321.2
5	90	304.3	18	4 750	322.5
6	150	304.6	19	5 500	325.2
7	250	305.0	20	6 500	328.4
8	500	306.1	21	7 500	331.0
9	750	307.8	22	8 500	334.9
10	1 000	309.5	23	9 500	338.7
11	1 350	311.8	24	10 500	342.5
12	1 750	314.2	25	11 500	345.7
13	2 250	316.4	26 ($\bar{s}$)	13 500	356.0

where

$$C_{kn} = SC'_{kn} \quad \text{and} \quad \lambda_j = S\lambda'_j. \tag{52}$$

According to (51) the eigenvectors do not vary in space, but the eigenvalues do vary by the factor S. In the practice of VMI it would be necessary to compute λ'_j and E_{nj} for the case $z_G = 0$ ($S = 1$), and then compute λ_j at each grid point from (52).

The other dependence on z_G is found in $\mathbf{C}^2$ and not in $\mathbf{C}^1$ or $\mathbf{C}^3$. That dependence resides in $\bar{\theta}$ which is an integral quantity. At all z^* levels below $\bar{s}^*$, θ varies systematically with z_G since the surfaces of constant z^* follow the terrain slope. The spatial dependence of $\mathbf{C}^2$ on $\bar{\theta}$ cannot be as easily evaded as that on S since $\bar{\theta}$ is not a simple multiplicative factor. Therefore, in the practice of VMI using (39), it would be necessary to compute $\mathbf{E}$ and its inverse at every grid point in the horizontal.

Eigenvectors and eigenvalues were computed for a 26-level version of the RAMMS using the values of z^* and θ_0 given in Table 1, with $z_G = 0$. The profile of θ_0 was derived from a July sounding at Denver, Colorado. The version of the RAMMS reported in Mahrer and Pielke (1977) has been initialized under the assumption that the lowest layer of the model is neutrally stratified. This condition is not suited to VMI because neutral stratification gives rise to vertical modes that do not correspond to propagating gravity waves. Accordingly, all layers were made stable for this experiment.

TABLE 2. Equivalent depths and phase speeds of vertical modes.

Mode	Case 1 H (m)	Case 1 c (m s^{-1})	Case 2 H (m)	Case 2 c (m s^{-1})	Case 3 H (m)	Case 3 c (m s^{-1})
1	796	88	12 176	345	12 223	346
2	86	29	159	39	151	38
3	33	18	41	20	41	20

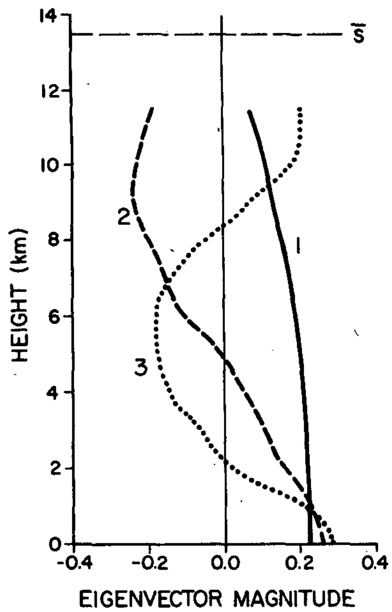

FIG. 1. The first three normal modes of the standard formulation of RAMMS, case 1.

The eigenvalues λ_i can be interpreted as gH_i, where H_i is the equivalent depth of the mode i. The phase speed c_i is given by $c_i = \sqrt{gH_i} = \sqrt{\lambda_i}$. Values of H_i and c_i for the first three modes are given in Table 2.

The first three vertical modes for case 1 are shown in Fig. 1, and for case 3 in Fig. 2 (those for case 2 were very similar to those for case 3). In case 1, the constraint that $\dot{\pi}(\bar{s}^*) = 0$ require that all modes go to zero at $\bar{s}^*$.

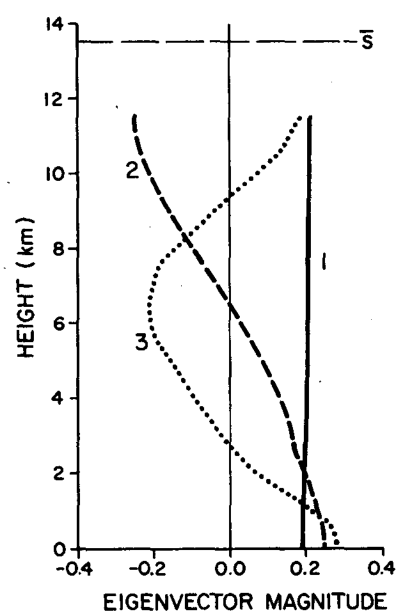

FIG. 2. The first three normal modes of the model formulation given in case 3.

4. Conclusions

A version of the Colorado State Regional Atmospheric Mesoscale Modeling System formulation has been adapted to a representation suitable for carrying out vertical mode initialization. The vertical modes of the standard formulation (case 1) show a fastest mode with a speed of about 90 m s^{-1}.

Results of this study indicate that it would be feasible to apply the BM version of VMI to a mesoscale model such as RAMMS using either $\dot{\pi}(s^*) = 0$ or $\dot{\pi}(\bar{s}^*) = 0$ as the top boundary condition on pressure. However, the feasibility of the $\dot{\pi}(s^*) = 0$ case depends strongly on the particular formulation used in the vertical structure equation. For both boundary conditions the phase speed of every mode will vary with surface elevation.

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