TURBULENCE CHARACTERISTICS ALONG SEVERAL TOWERS*

R. A. PIELKE and H. A. PANOFSKY
Dept. of Meteorology, College of Earth and Mineral Sciences, Pennsylvania State University, Pa., U.S.A.

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Abstract. Observations from several towers are used to show how measurements of wind and temperature near the surface can be used to estimate the variances of the horizontal velocity and the dissipation rate up to the top of the towers, provided the roughness length is known. The roughness length usually varies with wind direction, and the traditional methods of estimating it tend to lead to overestimates.

Analysis of cross spectra between velocity components at different levels shows that Davenport's Geometrical Similarity is satisfied. Coherence falls off exponentially with the ratio of height interval to wavelength, and the 'decay parameter' depends on Richardson number near the surface. Coherences at different sites show no significant differences in neutral air. The lateral velocity components have larger coherence and more time delay between levels than the horizontal components at all sites.

Time delay and coherence are also discussed in other Cartesian directions, and it is suggested that these quantities, having relatively simple properties, can be used as building blocks for an empirical three-dimensional model of turbulence.

1. Introduction

Wind fluctuations statistics and wind profiles have been measured on many towers. However, satisfactory analyses of the measurements have been rare, largely because of the complex terrain surrounding most of the towers. The relationships between wind fluctuations at different levels have been studied only by Davenport (1961), Shiotani (1968) and Panofsky and Singer (1965), often from very few combinations of levels.

The present paper will concentrate on observations at two towers, one at White Sands, New Mexico, the other near Cape Kennedy, Florida. Both towers are located in terrain which is relatively simple for several miles in some directions; and both towers are instrumented for wind measurements at many levels — six at Cape Kennedy and seven usable levels at White Sands. Temperature data are sufficient along both towers to describe well the vertical distribution of lapse rate.

The Meteorology Department at the Pennsylvania State University was given a grant by NASA, Huntsville, to analyze the winds at Cape Kennedy; and the Atmospheric Science Office, U.S. Army, made funds available to study the wind properties at White Sands. Comparable types of analysis had been carried out previously on observations taken on other towers with fewer instrumented levels, e.g., those at South Dartmouth, Mass. (instrumental systems and data handling under supervision of H. E. Cramer; some data reduction by Thomas Pries (U.S. Army Atmospheric Science Office)) and at Brookhaven, New York (instrumental systems and data handling under supervision of M. E. Smith). Turbulence properties at these towers will be
compared frequently with those at White Sands (to be abbreviated by WS) and Cape Kennedy (CK).

Also, an attempt will be made to generalize some of the mathematical description of the horizontal variation of turbulence characteristics, with the aim of eventually arriving at an empirical model of the four-dimensional space-time structure of atmospheric turbulence in the lowest 100 m or so. In order to make a start towards this objective, observations from other sites will be used also.

2. The Data

Table I lists the types of observations at the two principal towers as well as at the main two sites used for comparison. It is seen that only wind speed and direction were measured at WS and CK, and that vertical angles were only available at the comparison sites. Therefore, the characteristics of vertical motion are not analyzed in this paper. Also, temperature fluctuation statistics were obtained only at South Dartmouth.

| TABLE I  
<table>
<thead>
<tr>
<th>Sources of data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Cape Kennedy</td>
</tr>
<tr>
<td>Type of terrain</td>
</tr>
<tr>
<td>Height of fluctuation measurements (m)</td>
</tr>
<tr>
<td>No. of runs unstable, neutral</td>
</tr>
<tr>
<td>Fluctuation measurements</td>
</tr>
<tr>
<td>Type of sensors</td>
</tr>
<tr>
<td>Sampling rate for this paper (sec)</td>
</tr>
</tbody>
</table>

At CK and WS, the analysis of the observations was quite similar; each ‘period’ lasted about 1 hour. Wind components were determined in the mean wind direction ($u$), and at right angles to the wind direction ($v$). These components are referred to as ‘longitudinal’ and ‘lateral’, respectively. Lines of regression were fitted to the time
series of $u$ and $v$, and the deviations from these lines are the turbulent velocity components, denoted by $u'$ and $v'$. At the other two sites, the separation of fluctuations from means was accomplished by somewhat different methods (see, e.g., Busch and Panofsky, 1968).

3. Characterization of Stability and Roughness

In the surface layer, the Richardson number or the Monin-Obukhov ratio $z/L$ are frequently used to parameterize the hydrostatic stability, or, more exactly, the relative importance of mechanical turbulence and heat convection. The definitions of these two quantities are:

$$\text{Ri} = \frac{g}{T} (\gamma_d - \gamma) \left( \frac{\partial V}{\partial z} \right)^2$$

and

$$\frac{z}{L} = -\frac{kgHz}{u^*3c_p\rho T}.$$  

Here, $g$ is gravity, $z$ height, $T$ temperature, $\gamma$ the lapse rate, $\gamma_d$ the adiabatic lapse rate, $u^*$ the friction velocity, $H$ the vertical heat flux, $\rho$ the density, $k$ von Kármán's constant, $V$ the mean wind speed and $c_p$ the specific heat at constant pressure. The effect of moisture on $z/L$ and Ri is usually neglected. The von Kármán constant is near 0.4, and will be given numerically in later equations to avoid confusion with the wave number, also denoted by $k$.

The Richardson number is not a satisfactory parameter to judge the characteristics of turbulence up to 150 m, because, on convective days, it usually changes sign from negative below about 50 m to positive above, even though the heat flux remains upward. The quantity $z/L$ would be better, except that the heat flux was not available at WS and CK. Therefore, the quantity actually chosen as stability parameter was $z/L_0$, where $L_0$ is an estimate of $L$ near the ground. $L_0$ was obtained from the hypothesis, first suggested by Pandolfo (1966) that, in unstable air, near the surface, Ri and $z/L$ are essentially equal to each other. Thus, $L_0$ was estimated as the ratio of a low height $z$ to the Richardson number centered at that height. Only very few runs were obtained in stable air, and the same procedure was used in these cases, although the estimate of $L_0$ is then not good.

The roughness length was at first obtained according to a procedure suggested by Panofsky (1963). The winds were plotted as function of $\ln z - \psi(z/L_0)$. $\psi(z/L_0)$ is a 'universal' function, the various estimates of which agree surprisingly well with each other. Plotted on this scale, observations near the ground fall on a straight line, which intersects the ($\ln z - \psi$) axis at $\ln z_0$, where $z_0$ is the roughness length. This procedure was followed with the winds at CK and WS at the two lowest levels only, that is, 18 m and 30 m at CK, and 8 m and 22 m at WS. The roughness lengths determined in this manner at CK are listed in Table II, in the column labeled $\psi$-method. The variation of $z_0$ with wind direction is well correlated with terrain features in the various
TABLE II
Roughness lengths computed from wind profiles and spectra

<table>
<thead>
<tr>
<th>Run No.</th>
<th>Wind direction (degrees)</th>
<th>$z_0$, $\psi$-method, m</th>
<th>$z_0$, $\varepsilon$-method, m</th>
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</thead>
<tbody>
<tr>
<td>13</td>
<td>10</td>
<td>0.23</td>
<td>0.05</td>
</tr>
<tr>
<td>141</td>
<td>40</td>
<td>0.07</td>
<td>0.07</td>
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<tr>
<td>67</td>
<td>55</td>
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</tr>
<tr>
<td>30</td>
<td>70</td>
<td>0.12</td>
<td>0.02</td>
</tr>
<tr>
<td>165</td>
<td>100</td>
<td>0.38</td>
<td>0.14</td>
</tr>
<tr>
<td>101</td>
<td>105</td>
<td>0.42</td>
<td>0.39</td>
</tr>
<tr>
<td>163</td>
<td>110</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>176</td>
<td>115</td>
<td>0.17</td>
<td>0.16</td>
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<td>125</td>
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<td>133</td>
<td>140</td>
<td>0.40</td>
<td>0.14</td>
</tr>
<tr>
<td>162</td>
<td>180</td>
<td>0.80</td>
<td>0.35</td>
</tr>
<tr>
<td>155</td>
<td>180</td>
<td>0.90</td>
<td>0.40</td>
</tr>
<tr>
<td>142</td>
<td>210</td>
<td>0.10</td>
<td>0.02</td>
</tr>
<tr>
<td>91</td>
<td>275</td>
<td>0.48</td>
<td>0.33</td>
</tr>
<tr>
<td>121</td>
<td>335</td>
<td>0.26</td>
<td>0.26</td>
</tr>
</tbody>
</table>

directions from the CK tower: from NW to E, the terrain is relatively smooth, with some woods in the W and SW, and other obstacles to the south.

However, Blackadar et al. (1969) showed that, even as low as 30 m, the usual assumptions about the vertical variation of mixing length and stress are unsatisfactory, and that therefore the roughness lengths derived by the $\psi$-method are systematically too large. An entirely different method will be described later, which gives results listed in the column headed ‘$\varepsilon$-method’. These estimates are in better agreement with those derived with Blackadar’s correction, and will be assumed to be the best estimates of $z_0$.

The corresponding analysis at White Sands was much less extensive, since only a few wind profiles had large enough values of $|L_0|$ to permit an analysis by the first method. Further, only three runs were available giving satisfactory fast-response data. The roughness length in most directions is probably about the same (about 6 cm) since the vegetation is uniform. To the west of the tower, however, the ground has been smoothed so that the best estimate of $z_0$ in westerly winds appears to be 0.6 cm. The roughness lengths at South Dartmouth and Brookhaven have been discussed elsewhere (see, e.g., Panofsky and Townsend, 1964; Oliphant, 1964).

4. The Inertial Subrange of the Spectra

The spectra of the longitudinal and lateral wind components at CK have recently been analyzed and fitted to mathematical expressions by Fichtl and McVehil (1969). Here we shall concentrate only on the inertial subranges in these spectra, as well as those at WS, and the properties of energy dissipation, $\varepsilon$. As is well known, the spectra
of the lateral and longitudinal wind components (particular of the latter) follow the 
\(-\frac{3}{5}\) law to frequencies much lower than to be expected theoretically. Nevertheless,
these spectral regions can be used to determine the dissipation, \(\varepsilon\).

In practice, the spectra were multiplied by frequency, normalized by the squares of
the friction velocities based on the roughness lengths shown in the third column of
Table II, and machine-plotted as function of \(kz\) on bilogarithmic paper. (Here, \(k\)
is the one-dimensional wave number inferred from the frequency by use of Taylor’s
hypothesis.) A straight line with slope \(-\frac{3}{5}\) was fitted to the data by eye and inter-
polated for \(kz=1\). If the friction velocities used for the normalization had been
correct, this interpolated value would be proportional to \((\phi_e)^{2/3}\), where \(\phi_e\) is defined
as the ‘normalized’ dissipation,

\[
\phi_e \equiv 0.4z\varepsilon/u_0^*^3.
\]

Here \(u_0^*\) is the friction velocity at the surface. The constant of proportionality depends
on the ‘universal’ constants in the Kolmogorov law, which were taken as 0.5 for the
longitudinal wind components and 0.65 for the lateral components, for \(k\) in radians
per unit length.

Estimates of \(\phi_e\) from the spectra can be compared with values of \(\phi_e\) obtained from
a consideration of the budget of kinetic energy, which can be written, for equilibrium
conditions:

\[
\phi_e = (u^*/u_0^*)^3 \phi - (z/L_0) + C.
\]

Here \(\phi\) is the normalized wind shear, \((0.4 z/u^*)(\partial V/\partial z)\), and \(C\) is the normalized
energy flux convergence. According to Deardorff (1968), \(\phi\) may be represented by
\((1-18 z/L)^{-1/4}\) in unstable air in the surface layer. This expression particularly fits
Swinbank’s observations (1964) at Hay and Kerang which are regarded as the best so
far published.

There is considerable controversy about the behavior of \(C\). According to Lumley
and Panofsky (1964), Fichtl and McVehil (1969), and Deardorff (1968), this term is
negative near the ground in unstable air. Higher up it presumably passes through zero
and becomes positive at the top of the convective layer. It is here assumed that \(C\) can
be disregarded between 30 m and 150 m. In this height range we then put:

\[
\phi_e = (u^*/u_0^*)^3 (1 - 18z/L_0)^{-1/4} - z/L_0.
\]

The quantity \((u^*/u_0^*)^3\) differs from unity significantly only in the upper portion of the
layer under consideration, where the mechanical production is usually much smaller
than buoyant production. Therefore, this quantity can be put equal to unity as a first
approximation.

In order to test these various simplifications, the observed ratios of \((\phi_e)^{1/3}\) at 120
and 30 m for CK were plotted as function of \(z/L_0\) in Figure 1. On the same figure is
also plotted the corresponding ratio computed from Equation 5; the fit is quite good
and certainly does not suggest any systematic departures.

Next, for each individual run, \(\phi_e\) was plotted as function of height from 30 m to
150 m, and a curve of the form (5) with the given $L_0$ was compared to it. In almost all cases, such curves fitted well, provided all observed $\phi_e$ were multiplied by a constant. It should be recalled that the ‘observed’ $\phi_e$ had been obtained through normalization by friction velocities involving the first estimates of $z_0$. Forcing the ‘observed’ $\phi_e$ to fit these computed from Equation (5) implied new, and usually much lower values of $z_0$. These new estimates are given in the last column of Table II. This set of roughness lengths agrees fairly well with Blackadar’s estimates.

![Image](Fig. 2.0)

Fig. Ratio of $(\phi_e)^{1/3}$ at 120 and 30 m at Cape Kennedy as a function of $z/L_0$.

Incidentally, these results imply that a fair estimate of dissipation from 30 m to 150 m can be made from the equation:

$$
\varepsilon = \frac{u_0^*}{0.4z} \left[ (1 - 18 \frac{z}{L_0})^{-1/4} - \frac{z}{L_0} \right]
$$

where $u_0^*$ is given by (subscript 1 denotes a relatively low level)

$$
u_0^* = k V_1 \left[ \ln \frac{z_1}{z_0} - \psi \left( \frac{z_1}{L_0} \right) \right]^{-1}
$$

provided that $z_1$ is sufficiently small (less than 20 m or so) and that the roughness length is well known. According to Equation 6, $\varepsilon$ decreases about as $1/z$ in neutral air, but is essentially constant in unstable air except right near the surface. This result is not surprising since the various assumptions imply that $\varepsilon$ approaches $gH/c_p qT$ above the surface layer.

5. Estimation of Variance at Cape Kennedy

According to Monin-Obukhov similarity theory, the ratio of the standard deviations of the wind components to the friction velocity depends only on $z/L_0$. In the case of the $u$-components this dependence is quite weak, so that, as an approximation, we may put the standard deviations proportional to the friction velocities.

To test this hypothesis the standard deviation of $u$, $\sigma_u$, at 18 m was plotted as function of $u_0^*$, determined by Equation 7, making use of the roughness lengths in the last column of Table II. The result is shown in Figure 2. The scatter is quite small, and
the constant of proportionality, about 2.3, is within the range observed elsewhere (see Lumley and Panofsky, 1964). Thus \( \sigma_u \) at 18 m can be estimated given the roughness length, the Richardson number at relatively low levels, and the wind at 18 m. Similar estimates for \( \sigma_v \) appear less satisfactory (Figure 3).

In order to estimate standard deviations at higher levels, the equation of motion in the lowest 150 m can be written:

\[
\frac{\partial}{\partial z} u^* \sin \alpha = f V_g
\]

(8)

Here, \( \alpha \) is the angle between wind and isobars, and \( f \) the Coriolis parameter, and \( V_g \) the geostrophic wind speed. If we now assume that the rate of decrease of \( u^* \) with height is slow, that \( \sigma_u \) remains proportional to the local value of \( u^* \), and that \( V_g \) is approximately proportional to \( u_{18} \), we can write

\[
\sigma_{u_2}^2 - \sigma_{u_1}^2 = A u_{18}
\]

(9)

where \( A \) is a quantity which varies relatively little. Equation (9) was tested by plotting the difference between the variances of \( u \) at 150 and 18 m as function of \( u_{18} \), the wind speed at 18 m. The result is shown as Figure 4. Although the scatter is quite large, there is a definite correlation between the variance difference and the wind speed.

Fig. 2. The standard deviation \( \sigma_u \) at 18 m as function of surface \( u_0 \). Straight line fitted by eye.
Thus, the wind speed furnishes an estimate of velocity variance at higher tower levels, given an estimate of the variance near the surface. The corresponding computation with lateral velocity variances proved unsuccessful.

6. Coherence between Winds at Different Levels

Coherences* between u's and v's at different levels were available, under various stability conditions, from WS, CK, South Dartmouth and Brookhaven. In addition, Davenport (1967) and Shiotani (1968) have made measurements in strong winds when Richardson numbers were near zero. On the basis of his original analysis, Davenport (1961) formulated the hypothesis that the coherence between wind components at two heights, separated in the vertical by Δz, should be a function only of the ratio of Δz to λ, the horizontal wave length, provided Ri is near zero. This ratio will here be denoted by Δf, and computed through Taylor's hypothesis by:

\[ Δf = n Δz/\bar{V} \]  

(10)

* For definitions of coherence and similar terms, see Lumley and Panofsky (1964).
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Fig. 4. The difference between $\sigma_u^2$ at 150 and 18 m as a function of the average wind speed along the tower.

where $\bar{V}$ is again the mean wind speed in the layer $\Delta z$ and $n$ is the frequency. In general, we would expect the coherence to depend also on a dimensionless indicator of stability, such as $\text{Ri}$ or $z/L$.

At WS and CK, wind components at many levels were available, and coherences could be evaluated for a given run from many combinations of levels. For each run, all the results were computer-plotted as a function of $\Delta f$ on the same graph. A good example is Figure 5, which shows that the coherences for all pairs of levels fall essentially on the same line. There are no systematic departures as a function of height or spacing. Many other graphs show a great deal more random scatter than Figure 5, particularly for the lateral velocity components; but the scatter appears random, and Davenport's hypothesis of geometric similarity seems confirmed, at least between 8 and 150 m.

Another feature of the coherence-$\Delta f$ graphs was that they could be fitted by simple exponentials. We can therefore write:

$$\text{coh}(\Delta f) = \exp(-a \Delta f)$$  \hspace{1cm} (11)

$a$ will be called the 'decay parameter'.

Fig. 5. An example of a plot of coherence as a function of $df$. Line fitted by eye.

In Figures 6 and 7, values of $a$ from many sites for $u$ and $v$, respectively, are brought together. The abscissa was evaluated in different ways for the various sites, and the different estimates of $z/L_0$ are not strictly comparable. For example, at CK, $z$ was 23 m; at WS, 13 m. At Brookhaven, the stability was classified according to the appearance of the wind direction records into categories $B_1$, $B_2$, $C$ and $D$, as described by Singer and Smith (1953). Pries (1968) has given rough equivalences between these classes and simultaneously measured Richardson numbers between 11 m and 46 m. At most of the other sites, measurements had been made only in strong winds so that $z/L_0$ was set equal to zero. Thus it is possible to distinguish coherence properties with positive, near zero and negative $z/L_0$; more quantitative generalizations are premature.

Two lines were drawn on Figures 6 and 7 by eye: one fitting the observations at CK; the other, to fit all the other observations. The discrepancies between CK and the other sites in unstable air seem to be real, with smaller coherences for the same $|z/L_0|$ at CK. The reason is unknown. However, everywhere coherence increases with decreasing stability. At all places, the $v$-coherence in unstable air is greater than the $u$-coherence.

In the most important case of strong winds, there is little systematic disagreement between the characteristics of the coherence of wind speed in the vertical at different sites, as shown by Table III. Although there are differences, for example between Brookhaven and South Dartmouth, reference to Figure 6 will show that they are not
Fig. 6. The exponential decay constant $a$ of the longitudinal wind component as a function of $z/L_o$.

Fig. 7. The exponential decay constant $a$ of the lateral wind component as a function of $z/L_o$. 
significant. The best estimation $a$ for the $u$-component is about 19, with an uncertainty of perhaps ± 3. For the $v$-component, it is more nearly 14. These numbers imply that the coherence exceeds 50% only when the wavelength is 25 times the separation of the layer.

### TABLE III

<table>
<thead>
<tr>
<th>Location</th>
<th>$a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brookhaven</td>
<td>28</td>
</tr>
<tr>
<td>South Dartmouth</td>
<td>14</td>
</tr>
<tr>
<td>Cape Kennedy</td>
<td>20</td>
</tr>
<tr>
<td>White Sands</td>
<td>17</td>
</tr>
<tr>
<td>Nakagawa</td>
<td>15</td>
</tr>
<tr>
<td>Tokyo</td>
<td>20*</td>
</tr>
<tr>
<td>Sale</td>
<td>17*</td>
</tr>
<tr>
<td>Average</td>
<td>19</td>
</tr>
</tbody>
</table>

From Davenport (1967).

7. Slopes of the ‘Eddies’

The ratio of the quadrature spectrum to the cospectrum defines the phase lag between spectral components of a given frequency separated in space. Subject to Taylor’s hypothesis, then, an eddy ‘slope’ can be defined by

$$ s = \frac{1}{2\pi \Delta f} \arctan \left( \frac{\text{quad}(n)}{\text{co}(n)} \right) $$

where $\text{co}(n)$ is the cospectrum between the time series at the two levels, and $\text{quad}(n)$ the quadrature spectrum.

In atmospheric shear flow, ‘eddies’ tend to slope with the shear as first pointed out by Taylor (1958). Equation (12) defines the slope in such a way that a large $s$ implies a large departure from the vertical.

In practice, $s$ varies quite erratically, and large, apparently random differences occur from run to run and from frequency to frequency. It is certainly premature, in fact probably unnecessary, to postulate a systematic dependence of $s$ on $n$; for $s$ can be estimated only over a small range of $n$; for small $n$, the phase lag is poorly determined, and at large $n$, the coherence is too small for $s$ to have any meaning.

In spite of the large random variations, two characteristics of $s$ are outstanding: first, and most important, at all 4 sites, the slopes of the $v$-components are about twice as large as those of the $u$-components (see, for example, Figure 8). This means that, at a given tower, changes of wind speed at high levels are followed by similar changes farther down more rapidly than changes of wind direction. Apparently, ‘crosswind’ eddies are tilted by the shear more easily than eddies in the $x-z$ plane.

The second systematic effect is that the slopes tend to decrease with height; for
and McVehil (1969) have recently given mathematical expressions for spectra of $u$ and $v$ at CK. Unfortunately the mathematical characteristics of $u$ and $v$ differ from place to place, except in the Kolmogorov range; hence there will be a corresponding variation in quadrature spectra and cospectra. On the other hand, mathematical characteristics of spectra of the vertical velocity appear to be more universal.

8. Extension to Other Directions

There exist, so far, relatively few published measurements of cross spectra between time series of variables separated horizontally. But the data in existence suggest that Davenport Geometric Similarity is valid also in horizontal directions. Moreover, the coherences of like wind components fall off exponentially with increasing frequency. Thus, we may write, in general

$$\text{coh}_i(n) = \exp(-a_i A f_i)$$  \hspace{1cm} (15)

(no summation over $i$ is implied).

Here, $A f_i$ is now $n Ax_i / V$ where subscript $i$ would be 1 for longitudinal separation, 2 for lateral separation and 3 for vertical separation. Subscripts $j$ identify the velocity component in question.

The most complete study of the behavior of coherence with lateral separation is that by Shiotani (1969) based on anemometers at 40 m height on five masts during strong winds, with various angles between the anemometer line and the mean wind. His observations obey the equation:

$$\text{coh}_3(n) = \exp(-16 A f_2).$$ \hspace{1cm} (16)

The difference between this formula and that for vertical coherence in strong winds is probably not significant, so that it seems reasonable to assume that coherence in strong winds possesses cylindrical symmetry. Perhaps:

$$\text{coh}_i(n) = \exp(-17 A f_i)$$ \hspace{1cm} (17)

where $A f_i = n r / V$ and $r$ is the separation between anemometers in any direction at right angles to the wind. Observations at 2 m at O'Neill (see Haugen, 1959), agree with Equation (17) in unstable air. In stable air, coherence falls off much more rapidly. Armendariz and Lang (1968) also show that the coherence at a given frequency is largest in unstable air.

For lateral correlation, there is no phase lag, and $s$ can be taken as zero.

Haugen's tabulations also permit estimation of coherence in the longitudinal direction. In this direction, the decrease of the coherence of $u$ with frequency is much slower, with the decay constant perhaps half the constant in the lateral directions. Thus, the eddies are elongated into the $x$ direction, as pointed out earlier by Panofsky (1962). In the longitudinal direction, of course, there is a time delay. In fact, if Taylor's hypothesis is valid, the 'slope' $s$ is exactly one. This condition is satisfied with the O'Neill data.
Although there is a conspicuous lack of published correlation data in horizontal directions, additional observations should be available soon because of the importance of such data to problems involving vertically rising aircraft.

9. Properties of Space-Time Correlations

Many of the applications of turbulence statistics to missiles and vertically rising aircraft involve correlation functions with varying time and space coordinates. It is suggested here that the relatively simple properties of coherences and slopes in the Cartesian directions lead to relatively simple mathematical formulations for the correlation functions. Adding the cosine transform of Equation (13) to the sine transform of Equation (14), and assuming that the coherences decay exponentially, we may write:

\[ r_i(\Delta x_i, t) = \int_{0}^{\infty} \sqrt{F_i^{1}(n) F_i^{2}(n)} \exp(-a_i^1 \Delta f_i/2) \cos 2\pi n \left[ t - s(\Delta x_i/V) \right] \, dn \]

(18)

Here, \( F_i^{1} \) and \( F_i^{2} \) are normalized spectra of the \( j \)-component of the wind at positions 1 and 2 separated by \( \Delta x_i \).

If the separation is horizontal, we can often assume that \( F_1^1 = F_2^1 \), so that:

\[ r_i(\Delta x_i, t) = \int_{0}^{\infty} F_i^1(n) \exp(-a_i^1 \Delta f_i/2) \cos 2\pi n \left[ t - s(\Delta x_i/V) \right] \, dn \]

(19)

Thus, for example, the correlation function following the air with the mean wind speed is given by:

\[ R(t = s \Delta x/V) = \int_{0}^{\infty} F(n) \exp(-a_i^1 \Delta f_i/2) \, dn \]

(20)

Hence, space-time correlation functions between two time series can be obtained given only mathematical descriptions of their spectra, plus decay parameters and slopes. It is probable that the last two kinds of parameters depend on \( z/L \); whether they also depend on other factors such as terrain characteristics is not known.

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References


