The resolution of global spectral models

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The letter of Professor Pielke (1991) suggesting a definition of horizontal resolution of gridpoint models prompts me to reciprocate with a short essay on a corresponding analysis for global spectral models. The horizontal structure of dependent variables in spectral models is represented by series expansion of spherical harmonics Y_n^m, where $0 \leq |m| \leq n$ (e.g., Kubota 1959); the transform method (Machenhauer and Rasmussen 1972) is used to calculate nonlinear terms on a Gaussian latitude–longitude grid in all modern spectral models. Triangular truncation ($0 \leq n \leq N$) of the series is often used, as it offers uniform resolution on a spherical domain. As this note will try to show, there does not seem to be any straightforward way to define an equivalent mesh size for spectral models; this makes the estimation of effective resolution even more difficult than in gridpoint models.

A sometimes quoted estimate of spectral-model resolution consists in the average spacing between Gaussian latitudes of the transform grid; for triangular truncation, this spacing is equal to that between longitudes at the equator: $L_1 \approx 2\pi a/(3N + 1)$, with a the radius of the earth. Hence, $L_1 \approx 13.3/N$ in units of thousands of kilometers; for a T31 model, $L_1 = 426$ km. This estimate of resolution is overly optimistic because the dimensions of the transform grid are chosen to allow calculation of quadratic terms without aliasing of the resolved spectral fields, and hence the transform grid is finer than required by the information content of the corresponding spectral series.

A more realistic estimate of resolution is given by the size of half a wavelength of the shortest resolved zonal wave at the equator: $L_2 = \pi a/N \approx 20/N$ in units of thousands of kilometers. Hence, a T31 model would have a resolution of $L_2 = 646$ km, according to this measure. Triangular truncation provides an isotropic and uniform resolution on a sphere. The shortest resolved zonal wave ($|m| = N$) used to determine L_2 corresponds to a mode with the gravest meridional structure, because modes that are very short in one direction must be elongated in the other direction: sectorial spherical harmonics ($|m| = n$) have modal structures shaped as orange segments and have their largest amplitude in the tropics, hardly an adequate measure of resolution for general circulation models.

An alternative way to estimate the resolution of spectral models is as follows. Consider that the area of the earth's surface is given by $4\pi a^2$; there are $(N + 1)^2$ real coefficients to a spherical harmonic series at triangular truncation with maximum index N, that is, $N(N + 1)/2$ complex coefficients for the modes $1 \leq |m| \leq N$, plus $(N + 1)$ real coefficients for the modes with $m = 0$. If an equal area on the surface of the earth is assigned to every piece of information contained in the series, this gives a footprint of surface area equal to $4\pi a^2/(N + 1)^2$ for every real coefficient. Resolution in a spectral model could be defined as the width L_3 of a flat, rectangular tile of the same surface area: $L_3 = (4\pi)^{1/2} a/(N + 1) \approx 22.6/N$ in units of thousands of kilometers. Hence, a T31 model would have a resolution of $L_3 = 728$ km, according to this measure.

Yet another definition of resolution for spectral models would be to consider the representative spatial dimension of high-order tesseral harmonics ($0 < |m| < n = N$). The eigenvalue of the Laplacian operator applied on a spherical harmonics Y_n^m is $-K^2 \equiv -n(n + 1)/a^2$. Equating the eigenvalue of the highest resolved mode with the corresponding eigenvalue of Fourier modes in Cartesian geometry for the purpose of estimating resolution gives: $K^2 = N(N + 1)/a^2 = k_x^2 + k_y^2$. Considering modes with unity aspect ratio, $k_x^2 = k_y^2 = k^2$, that is, with checkerboard-like modal structure, gives $k^2 = N(N + 1)/(2a^2)$. An alternative measure of resolution would be one-half of the corresponding wave-length: $L_4 \equiv \pi/k \approx 2^{1/2}\pi a/N \approx 28.3/N$ in units of thousands of kilometers. So for a T31 model, $L_4 = 899$ km.

It is noteworthy that the estimates of resolution given by L_2, L_3, and L_4 are all coarser than the simple-minded, overly optimistic estimate given by L_1. Pielke's

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remarks about the "effective" resolution of finite-difference models being coarser than the mesh size also apply to spectral models to some extent. It would be naive to think that the effective resolution of a spectral model is defined by even the most pessimistic of the estimates suggested above. Even though the Galerkin formalism of spectral models removes the numerical approximations associated with the horizontal discretization in finite-difference models, time-discretization errors and aliasing in the calculation of some of the terms, especially in the parameterization of physical effects, are unavoidable. Also, the horizontal dissipation that is applied to the upper part of the spectrum to prevent "spectral blocking" effectively reduces the information content below its theoretical limit given by the spectral truncation. For example, even the most scale-selective formulation suggested by Leith (e.g., Boer et al. 1984) substantially damps the upper 20% part of the resolved spectrum in low-resolution versions; other formulations, such as harmonic or biharmonic diffusion, are even less scale selective. Therefore any of the aforementioned definitions of L must be viewed simply as upper limits to the effective resolution of a spectral model for a given truncation.

References

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