Nonlinear or Linear; Hydrostatic or Nonhydrostatic Mesoscale Dynamics?

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Goal

To study under which conditions and to what extent **the hydrostatic approximation** and the **linearization of advection** hold in the mesoscale atmosphere, forced with a wide range of temporal and spatial scales

Applications

Hydrostatics

- Important for remote sensing
- Nonhydrostatic component can be recovered (Song et al., 1985)

Linear Advection

- Linear solutions are exact and run faster
- The nonlinear part *only* can be integrated numerically and linear solved exactly (Wiedman and Pielke, 1983)

Methodology I

The only previous study which explores these issues over a wide range of temporal and spatial scales is Dalu et al. (2003) which analyzed the problem following this procedure:

- reduce the set of equations shown to the left to the Poisson equation for the geopotential defined as $\phi=\theta_0\pi$, where π is the perturbation Exner function
- analyze the differences
  - linear – nonlinear
  - hydrostatic - nonhydrostatic

$$\left(\frac{\partial}{\partial t}+\lambda\right)u+U\frac{\partial u}{\partial x}-fv+\frac{\partial\phi}{\partial x}=K\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)u$$

$$\left(\frac{\partial}{\partial t}+\lambda\right)v+U\frac{\partial v}{\partial x}+fu=K\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)v$$

$$\left(\frac{\partial}{\partial t}+\lambda\right)w+U\frac{\partial w}{\partial x}+\frac{\partial\phi}{\partial z}-b=K\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)w$$

$$\left(\frac{\partial}{\partial t}+\lambda\right)b+U\frac{\partial b}{\partial x}+N^2w=Q+K\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)b$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0$$

$$b = g\,\frac{\theta'}{\theta_0}$$

Constant Coriolis

Constant eddy viscosity

Rayleigh friction

Background wind, replaced by u for the nonlinear advection case

Incompressibility

Prescribed flux

Vertical advection of (θo + θ(z))

Dalu et al. 2003 main conclusions:

- Periodic forcing case, $Q=Q_0\exp(-\mu_0 z)\cos(\omega t+kx)$:
  - There are two regimes, similarly to Rotunno (1983) transition depends on the vertical and horizontal scale of the forcing
- Hydrostatics:
  - Hydrostatic approximation generally valid unless: $\omega \approx N$ or $H/L \approx 1$ ϕ_d is significant
  - Song et al correction works almost everywhere
- Linear terms
  - ϕ_d is important where momentum div ≈ heat flux vert div
  - ϕ_b is not negligible when adv time ≈ forcing period
- Nonlinear terms
  - $\delta\phi_d$ significant if ω/(adv time) ≈ N^2
  - $\delta\phi_b$ must be considered when ω^{-2} ≈ (adv time)^2

ϕ_b geopotential component due to the vertical convergence of buoyancy

ϕ_d "dynamical pressure" component of the geopotential

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Methodology II

We specifically modified the Regional Atmospheric Modeling System 4.3 (RAMS) not only to simulate the same equation set as in Dalu et al. (2003), but also to extend the theory to a more realistic atmosphere, by relaxing some of the assumptions made in the mentioned study and retaining the additional terms which arise from these changes or simply diagnosing the eventual differences (ie the Exner function tendency (see table below).

	Equation	Additional Terms
u-mom Dalu et al	$\left(\frac{\partial}{\partial t}+\lambda\right)u+U\frac{\partial u}{\partial x}-fv+\frac{\partial\phi}{\partial x}=K\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)u$	
u-mon RAMS	$\left(\frac{\partial}{\partial t}+\lambda\right)u+U\frac{\partial u}{\partial x}-fv+\vartheta_0\frac{\partial\pi}{\partial x}=K\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)u$	$-(\vartheta(z)+\vartheta')\frac{\partial\pi'}{\partial x}$
w-mom Dalu et al	$\left(\frac{\partial}{\partial t}+\lambda\right)w+U\frac{\partial w}{\partial x}+\frac{\partial\phi}{\partial z}-b=K\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)w$	
w-mom RAMS	$\left(\frac{\partial}{\partial t}+\lambda\right)w+U\frac{\partial w}{\partial x}+\theta_0\frac{\partial\pi}{\partial z}-g\frac{\theta'}{\theta_0}=K\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)w$	$-\vartheta'\frac{\partial\pi'}{\partial z}+\vartheta(z)\frac{\partial\pi}{\partial z}$
b Dalu et al	$\left(\frac{\partial}{\partial t}+\lambda\right)b+u\frac{\partial b}{\partial x}+N^2w=Q+K\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)b$	
θ *RAMS*	$\left(\left(\frac{\partial}{\partial t}-\lambda\right)\theta+u\frac{\partial\theta}{\partial x}+w\frac{\partial\theta(z)}{\partial z}=F+K\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\theta\right)\frac{g}{\theta_0}$	$-w\frac{\partial\theta'}{\partial z}\frac{g}{\theta_0}$
Continuity Dalu et al	$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0$	
Continuity RAMS	$\frac{\partial\pi'}{\partial t}=-\frac{R\pi_0}{C_v\rho_0\theta_0}\left(\frac{\partial\rho_0\theta_0 u}{\partial x}+\frac{\partial\rho_0\theta_0 w}{\partial z}\right)$	

Issues

After several thorough revisions of the code, and a very careful tuning our version of RAMS, when initialized with the analytical solution the model quickly rearranges the pressure field. This problem seems to indicate that the several changes made to the dynamical core are not compatible with the semi-implicit scheme which computes the vertical velocities.

Preliminary Results

Regardless of the above difficulty, the individual terms of the Poisson equation for the geopotential were diagnosed for a periodic forcing (1 h in time, 10 km in space) which decay vertically with e-folding length of 1000 m. Consistently with Dalu et al. the two major contributions to the change in the Laplacian of the geopotential from linear to non linear advection are due the dynamical pressure and the vertical gradient of buoyancy (see figure below).

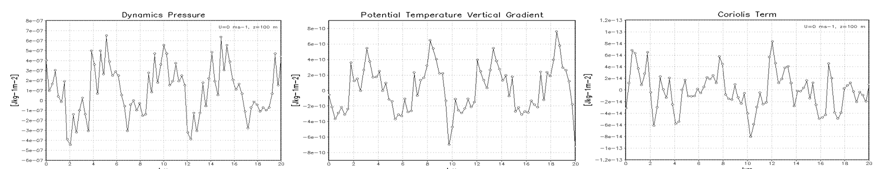

Fig. 1 Contributions to the geopotential difference between the nonlinear and linear cases for L_x = 10 km, T= 1h, U=2.5 ms^{-1} at z = 100 m. The units are those of the geopotential Laplacian. The absolute values are small since the typical length of the Exner function variations L_x, so that the Laplacian scales as $\Delta\phi/L_x^2$. The Coriolis term is several order of magnitudes smaller than the others.

References

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