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Reply to Jascourt and Raymond

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Jascourt and Raymond (1992; henceforth denoted by JR) raise some valid points regarding our paper "Chaos in Daisyworld" (Zeng et al., 1990; henceforth denoted by ZPE). However, much of their criticism is based on simple misunderstandings, and we welcome the opportunity to clarify these issues.

We agree with JR that the discrete model of Daisyworld is different than the differential model, and we discussed this point in detail throughout ZPE. We referred to both as Daisyworld, since we considered them to be two different models of the same physical system in which daisies interact strongly with their environment. To motivate our discrete model, we first described the differential model of Watson and Lovelock (1983; henceforth denoted by WL), after which we described our discrete model and discussed differences between the two. The latter was proposed as an alternative model of Daisyworld, not as a mathematical approximation to the differential model of WL. We regret that this point was not as clear as we had intended, and we agree with JR that our notation in eq. (9) was partly to blame. In particular, after we explained the selection of $\Delta t = 1$, we should have rewritten eq. (9) as

$$a_i^{n+1} = a_i^n + a_i^n (x^n \beta_i^n - \gamma_i^n). \tag{1}$$

Also, we agree with JR that the sentence they refer to should have been worded differently as: "We start with the model of [WL], but write the equations in a more general way." However, we did discuss this difference several times throughout the paper.

With regard to the question of whether the discrete model or the differential model is more appropriate, we refer to Carter and Prince (1981). While they do write down a differential equation, to which WL refer as the basis for their differential model, Carter and Prince (1981) also conduct an experiment and show that the experimental data actually agree closely with the discrete equation similar to eq. (1) above (see their Fig. 1). The main problems with the differential model are that it allows an instantaneous response to any change, and it does not allow extinction to occur in a finite time. In contrast, the discrete model avoids both of these drawbacks. Furthermore, the discrete model with a finite generation time is a first step toward including seasonal variation, since it synchronizes the birth at the beginning of each new generation, which corresponds to the beginning of the growing season, rather than allowing continuous birth throughout the year.

As a related point, it must be understood that the discrete equations are valid only when $a_i \geqslant 0$. If a_i becomes zero, extinction has occurred, after which it remains zero. It is meaningless to continue using the discrete equations and allow a_i to become negative. Hence, the unphysical behavior referred to in JR does not really occur—JR have simply neglected to recognize the occurrence of extinction. Such cases were not included in ZPE because we considered cases of extinction to be irrelevant to the conclusions of that paper.

In order to discuss the coupling constant C and the generation time Δt, we rewrite eq. (1) above as

$$\begin{aligned} a_i^{n+1} &= a_i^n + a_i^n (x^n \beta_i^n - \gamma_i^n)\,\Delta t \\ &= a_i^n + a_i^n [x^n(\beta_i^n/\lambda) - (\gamma_i^n/\lambda)](\lambda\,\Delta t), \end{aligned} \tag{2}$$

which shows that changing Δt is equivalent to changing both the growth rate β_i and the death rate γ_i. This simply illustrates that the numerical values of these rates depend on the unit of time chosen for Δt. Physically, the generation time is nearly fixed: unlike the growth rate, it changes only slightly as the climate changes. Hence, in ZPE, this generation time was chosen as the unit of time (i.e., we measured time in generations), leading to eq. (1) above. We then investigated the effect of varying the growth rate β_i by varying its coupling to the local temperature (as given by the coupling constant C). Of course, one can mathematically vary Δt as well, as is done in JR, but this is not physically meaningful, since the generation time Δt is not physically variable.

As mentioned above, we have discussed the difference between the discrete model and the differential model in detail throughout ZPE. It is known from basic chaos theory, even without any computations, that, for differential Daisyworld, chaos can never occur in the one- and two-species cases. This is verified in WL and in ZPE. It is further shown in ZPE that steady-state behavior is obtained in differential Daisyworld with more than two species for a wide range of parameters. In contrast, for discrete Daisyworld, it is shown in ZPE that periodic and even chaotic oscillations can occur in the one- and two-species cases.

When eq. (1) is used, it is shown in May (1976) and by ZPE that chaos is possible when the temperature feedback is excluded. Chaos is also possible when the feedback is included, as shown in ZPE. It is further shown in Subsection 3.2.2 of ZPE that, in Daisyworld with only black daisies, when environmental feedback is included, chaos occurs for some parameter values for which chaos does not occur without the environmental feedback. Therefore, it is found in ZPE that the remarks made by Lovelock (1986) that the environmental feedback appears to stabilize the system, are not true in general.

It is correct, as stated in JR, that the main source of chaos is the delay in the response of the daisy population. When the coupling to the environment is large enough, the daisies cannot respond quickly enough to achieve equilibrium. Hence, chaos can occur in the discrete model either with or without environmental feedback. However, it is also true that adding such feedback does not necessarily lead to stability in Daisyworld, as claimed in WL based on the differential model, and this is the main point of ZPE.

The time-delay differential equations discussed in JR represent yet a third model, which is currently being investigated by several researchers. However, since this model is not discussed in ZPE, we will not pursue it here, except to say that it is another reasonable model of Daisyworld, and that chaos occurs in this model as well.

It is shown by the solid line and dark shading of Fig. 1 in JR that the long-term average of a chaotic state in discrete Daisyworld is close to the equilibrium solution of differential Daisyworld. It is well known that, for a chaotic attractor, the chaotic state always oscillates irregularly about the equilibrium state, and that the long-term average is always close to this equilibrium state. Since it can be seen from eqs. (1) and (9) in ZPE that the equilibrium solutions for differential and discrete Daisyworld are the same, the above conclusion of JR is simply a special case of this general statement, and, of course, it is correct. However, this illustrates that Daisyworld is unstable, rather than stable, since it fails to achieve equilibrium. While the lightly shaded regions of Fig. 1 in JR do show that the presence of the daisies reduces the variation of the effective temperature with luminosity, it does not demonstrate stability. For a given luminosity, the variation of the effective temperature with time is still highly chaotic with large-amplitude fluctuations, as was discussed in ZPE.

In summary, the conclusions of ZPE are correct and physically relevant. Some of the comments in JR are valid, but others are based on misunderstandings. Both works raise important questions regarding the validity and interpretation of the Gaia hypothesis and point out the need for further study.

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