NOTES AND CORRESPONDENCE

Long-Term Variability of Climate

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1. Introduction

In this research note, we address the following general question: In a nonlinear dynamical system (such as the climate system), can a known short-periodic variation lead to *significant* long-term variability? It is known from chaos studies (e.g., Lorenz 1991) that any perturbations in chaotic dynamic systems can lead to a red-noise spectrum; however, whether a *significant* long-term variability can be induced is unknown. To perform this study, an idealized nonlinear model developed by Lorenz (1984, 1990) is used. The model and the results are presented in sections 2 and 3, respectively. Finally, the implications of our research to the understanding of the natural variability of the climate system due to internal dynamics will be discussed in section 4.

2. Model

The model used in this study is described in Lorenz (1984, 1990). The model uses these three equations:

$$\frac{dX}{dt} = -Y^2 - Z^2 - aX + aF, \quad (1)$$

$$\frac{dY}{dt} = XY - bXZ - Y + G, \quad (2)$$

$$\frac{dZ}{dt} = bXY + XZ - Z, \quad (3)$$

where, as suggested by Lorenz, X defines the strength of a large-scale westerly wind and, through the thermal wind relationship, the geostrophically equivalent large-scale poleward thickness gradient. Lorenz scaled the variables such that two additional coefficients, which otherwise appear, are reduced to unity. The dependent variables Y and Z represent the strengths of the cosine and sine phases of a series of superposed waves. The rate of poleward heat transport is proportional to the square of the amplitude of Y and Z. The cross-product terms XY and XZ describe the amplification of the waves, except when X becomes negative, through their interaction with the westerly large-scale flow. The resultant loss of energy to the westerly flow is represented by the Y^2 and the Z^2 terms. The terms bXY and bXZ represent the displacement of the eddies by the westerly current. The deceleration of the westerly flow and waves by drag and thermal damping is represented by the linear terms. The damping time of 5 days for the eddies is chosen as the time unit. The values of F and G are specified and represent symmetric and asymmetric thermal forcing.

This paper applies this model to evaluate the natural internal irregularity of the system for 400 000 days (or about 1100 years) when no external physical mechanisms are included. The Lorenz papers provide an effective discussion of the nonlinear dynamics of the system, but those papers do not explore the significance of the results with respect to long time periods.

3. Results

Equations (1)–(3) are solved using the fourth-order Runge–Kutta algorithm. The values of $a = 0.25$, $b = 4$, $G = 1.0$, and $\Delta t = 0.025$ units (which corresponds to 3 hours) were the same as those used by Lorenz (1990). The initial condition was specified as $(X, Y, Z) = (2.0, 1.0, 0)$. Two forms of F value are specified:

$$F = 8.0, \quad (4)$$

$$F = 7.0 + 2.0 \cos(2\pi t/\tau), \quad (5)$$

where $\tau = 12$ months. The situation in which $F = 8.0$ corresponds to a perpetual winter condition. The time-varying form of F was used to represent an annual cycle of cross-latitude thickness gradients. In both cases, chaotic responses are reported in Lorenz (1990).

The model was integrated for 1101 years (each year is 365 days), and the data for the first year were deleted to avoid the influence of transients on the subsequent

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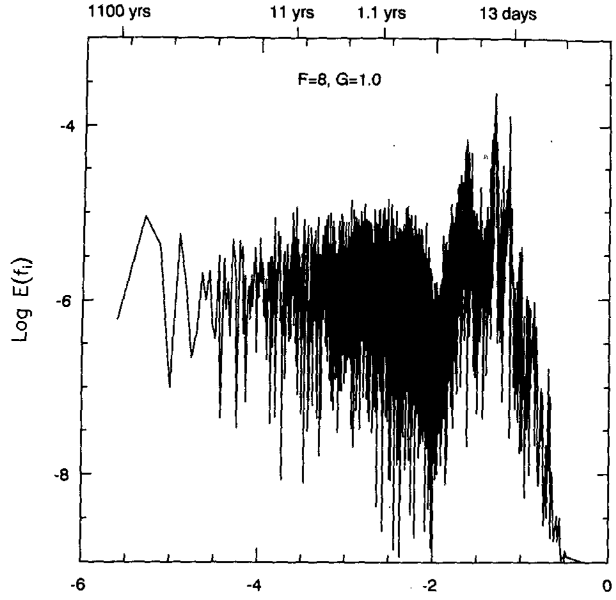

FIG. 1. Logarithm of power spectrum $E(f_i)$ as a function of the logarithm of frequency f_i computed from $N = 400\,000$ data of X for $F = 8.0$. Several values of the corresponding period p_i, which are used in Table 1, are given at the top of the figure. The initial values are $(X, Y, Z) = (2.0, 1.0, 0.0)$.

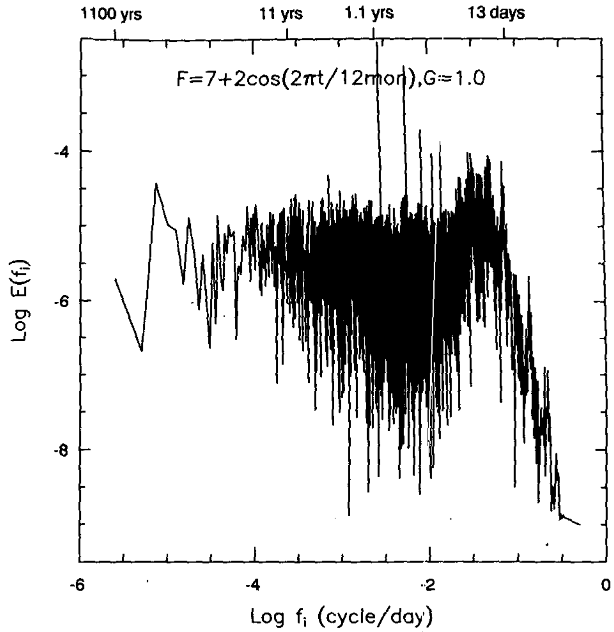

FIG. 2. Same as Fig. 1 except that F varies as in Eq. (5).

power spectrum analysis. The values of X after the first year were stored every eight time steps (i.e., once a day) for $N = 400\,000$ days (or about 1100 years). Power spectrum for the daily time series, X_i, was calculated by means of a fast Fourier transformation (FFT) for the two cases of Eqs. (4) and (5), and results are presented in Figs. 1 and 2. For the daily time series X_i ($i = 1, 2, \cdots, N$) with mean value $\bar{X} = \sum_{i=1}^{N} X_i / N$ and the variance $\sigma_X^2 = \sum_{i=1}^{N} (X_i - \bar{X})^2/N$, the power spectrum $E(f_i)$, as a function of frequency f_i, has the property that

$$\sum_{i=1}^{N/2} E(f_i) = \sigma_X^2, \quad (6)$$

where the frequency $f_i = i/N$ is in units of cycles per day. The period of any value in Figs. 1 and 2 is given by $p_i = N/i$, and several values of the period p_i are plotted at the top of both figures. Broadband noise in these figures is a qualitative indicator that chaos occurs.

For $F = 8.0$, the mean value $\bar{E}$ and the root-mean-square difference σ_E of $E(f_i)$ are given in Table 1. It is seen that at the longer time periods the variability is smaller in amplitude such that, according to this model, most of the temporal variation in the intensity of global circulation occurs at shorter time periods, with less variability in magnitude at the longer time scales of centuries.

For the situation with a seasonally varying value of F, it is seen from Fig. 2 that the largest contributions to time variations are from the annual cycle and its harmonics with periods of 1/2, 1/3, 1/4, and 1/5 years. However, these variations are predictable since the annual cycle is imposed in the model. The mean value $\bar{E}$ and the root-mean-square difference σ_E of $E(f_i)$ for Fig. 2 are shown in Table 1. It is seen that the variability of periods from 1.1 to 1100 years is smaller than that within a year. In an effort to minimize the effects of predictable seasonal variations of X after integrating the model for 400 000 days, the mean value for each day is computed by averaging X_i over the record for that day of every year. These daily means are then subtracted from each daily value. The corresponding

TABLE 1. The mean value $\bar{E}$ and the root-mean-square difference σ_E of $E(f_i)$ for different periods in Figs. 1 to 3: periods of 11 to 1100 years, periods of 1.1 to 11 years, and periods of 13 days to 1.1 years. The variance σ_X^2 in Eq. (6) is also given.

	Figure 1	Figure 2	Figure 3
σ_X^2	0.349	0.385	0.360
		$\bar{E}$	
11 to 1100 yr	7.51×10^{-7}	2.28×10^{-6}	2.29×10^{-6}
1.1 to 11 yr	7.50×10^{-7}	1.69×10^{-6}	1.69×10^{-6}
13 d to 1.1 yr	1.08×10^{-6}	2.30×10^{-6}	1.64×10^{-6}
		σ_E	
11 to 1100 yr	7.29×10^{-7}	2.02×10^{-6}	2.02×10^{-6}
1.1 to 11 yr	7.65×10^{-7}	1.82×10^{-6}	1.82×10^{-6}
13 d to 1.1 yr	3.67×10^{-6}	2.13×10^{-5}	2.65×10^{-6}

power spectra are given in Fig. 3. It is seen that the annual cycle and its harmonics disappear as expected. The mean value $\bar{E}$ and the root-mean-square difference σ_E of $E(f_i)$ for Fig. 3 are given in Table 1. It is seen from Table 1 that, after removing the predictable seasonal variations in the model output, variability at decade to century time scales is larger than the interannual variability and the variability within a year. It is also seen from Figs. 1 to 3 that red-noise spectra are obtained with the deterministic Lorenz model; in contrast, red-noise spectrum can also be obtained from a white-noise forcing in stochastic climate models (Hasselmann 1976; Frankignoul and Hasselmann 1977; Lemke 1977).

In order to establish the statistical significance of our results, we have also performed an ensemble of experiments with 20 different initial conditions (i.e., $X = 0.5, 0.6, \cdots, 2.4$, and $Y = 1$ and $Z = 0$, as in the above experiment). Figures 4–6 show the results of the 20 different experiments for the cases of the constant F, the time-dependent F, and the time-dependent F with the annual cycle removed before the power spectrum analysis, respectively. It is seen from Figs. 4–6 that $\bar{E}$ and σ_E change little with different initial conditions. In other words, our results discussed above are statistically significant. Therefore, using the Lorenz model, we have demonstrated that short-periodic variation can lead to substantial long-term variability in a nonlinear system.

In summary, this paper uses an idealized model developed by Lorenz (1990) to explore the long-term fluctuations of the model's climate. While the model is a very simplified system, Lorenz used it successfully to investigate chaos and intransitivity as related to interannual variability. Lorenz demonstrated that chaos in winter and intransitivity in summer can lead to interannual variability due to wave–mean flow interactions. Our paper extends this work and shows that the seasonal cycle itself can generate chaos in winter and intransitivity in summer due to wave–mean flow interactions, and that chaos and intransitivity then are sufficient to generate decadal and century time scale variations in the model's climate that are on the same order as interannual variations. No long-term physical mechanisms are required to account for substantial long-term deviations in climate as represented by this model.

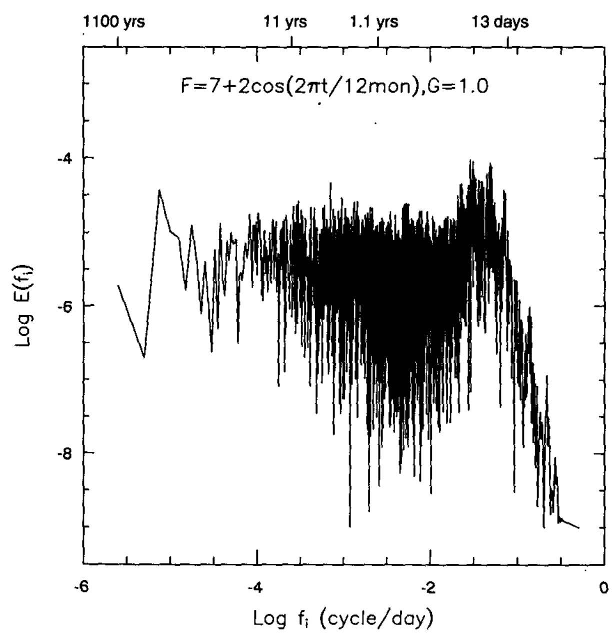

FIG. 3. Same as Fig. 2 except that X' (which is X minus the daily mean values), rather than X, is used to compute the power spectrum so that the predictable annual cycle is removed in the model output.

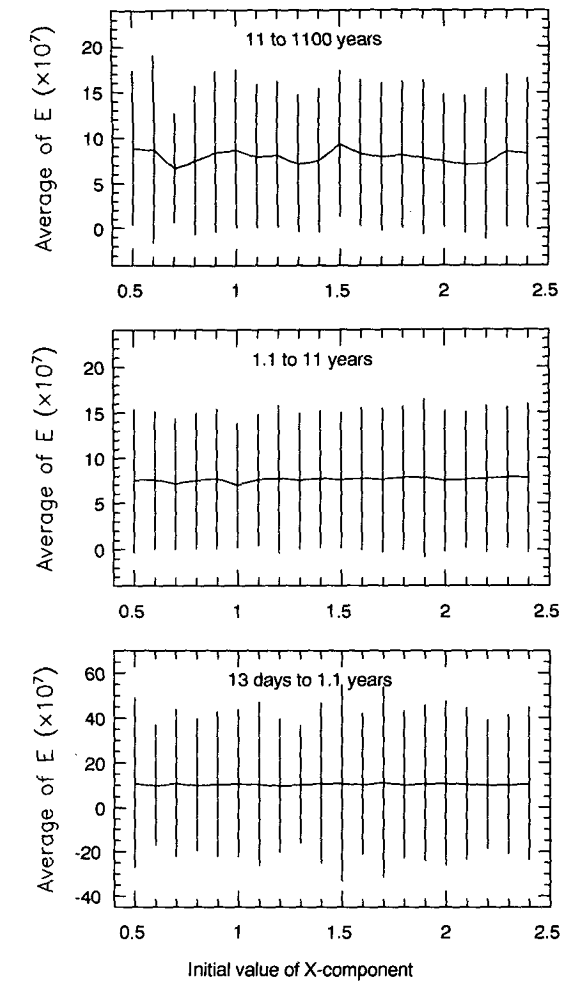

FIG. 4. The mean value $\bar{E}$ of $E(f_i)$ for $F = 8.0$ for different periods (periods of 11 to 1100 years, periods of 1.1 to 11 years, and periods of 13 days to 1.1 years) with 20 different initial values of X ($X = 0.5, 0.6, \cdots, 2.4$). The initial values of Y and Z are 1.0 and 0.0, respectively. The vertical lines represent the root-mean-square difference σ_E.

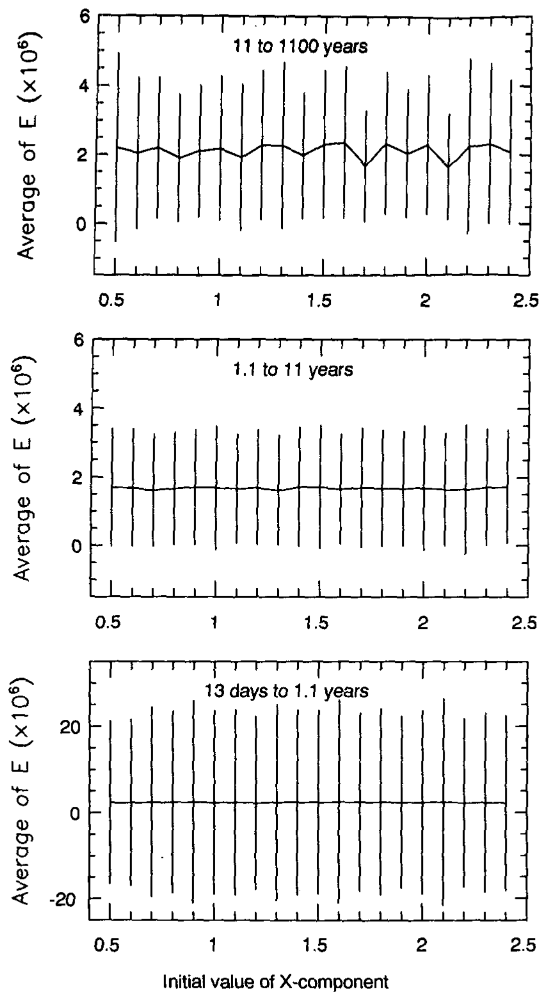

FIG. 5. Same as Fig. 4 except that F varies as in Eq. (5).

4. Implications of model results

One of the crucial problems in assessing man-made climate change is to understand the natural variability of the climate system and the cause of this variability. The interannual variability can be caused by external (non-man-made) forcing (e.g., a volcanic eruption) and the coupling of the climate system among the atmosphere, ocean, and biosphere (such as the air–sea interactions for an ENSO event). A natural variability of the atmosphere over decadal time scales can be caused by external forcing (e.g., the 22-year sunspot cycle) and the slow changes of other components of the climate system (such as the ocean thermohaline circulation). On the time scales of tens of thousands of years, natural variability can be generated by external forcing (e.g., orbital change). In general, deterministic nonlinear interactions (Lorenz 1991) and stochastic forcing (Hasselmann 1976) can also be used to explain the natural variability of different time scales. However, it is not very clear what the main reasons are for the natural variability of century and millennium time scales.

General circulation model (GCM) integrations can be applied to quantitatively investigate the intrinsic variability of climate over annual, decadal, and century time scales; however, to perform such a study (especially for century time scales), the GCMs must include the coupling of the atmosphere, ocean, and biosphere. Hansen et al. (1990) used a GCM with an atmosphere linked to a simplistic ocean model to explore this variability. More recently, several groups have integrated coupled ocean–atmosphere GCMs for as long as 100 years to study the transient responses of GCMs to a gradual change of atmospheric CO_2 (e.g., Manabe et al. 1991; Manabe et al. 1992; Barnett et al. 1992). James and James (1989, 1992) investigated the ultra-low-frequency variability (ULFV), that is, the natural variability between several years and several decades, in a simple atmospheric circulation model. By means

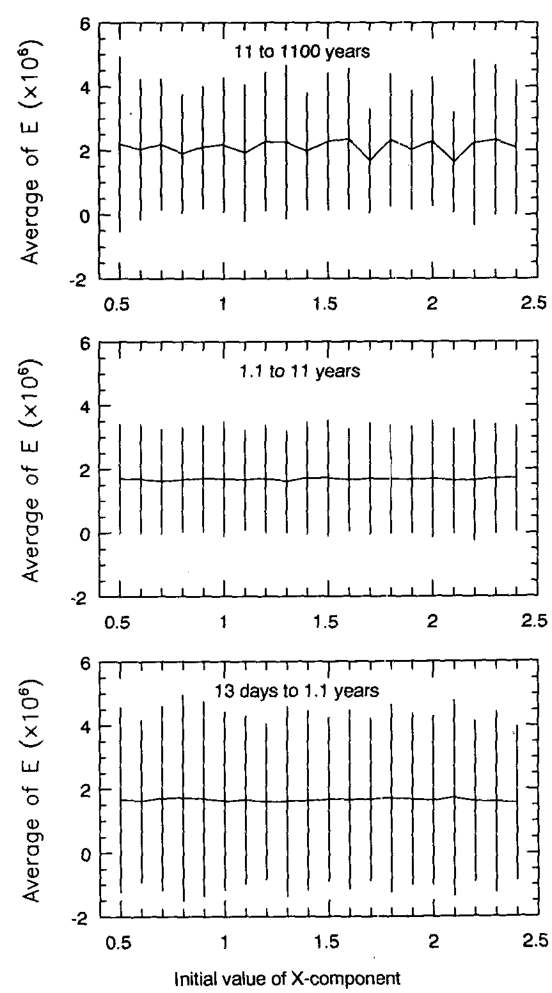

FIG. 6. Same as Fig. 5 except that X' (which is X minus the daily mean value), rather than X, is used to compute the power spectrum so that the predictable annual cycle is removed in the model output.

of an empirical orthogonal function analysis, they found that the principal component of the maximum ULFV (on time scales of around a decade) is an alternate splitting and coalescence of the subtropical and midlatitude jets. Analyzing the sea level air temperatures averaged over the globe and over one-year intervals from a simple model of 27 variables, Lorenz (1991) mentioned the upward and downward trends lasting through one or several decades that are due to cloud–albedo feedback process. Frankignoul and Hasselmann (1977) demonstrated, using a stochastic Budyko–Sellers model, that the white-noise spectrum (representing short-period "weather" disturbances) can produce a red spectrum, with most of the variance concentrated in very long periods. Using a stochastical climate model, Lemke (1977) also showed that surface temperature variability over the range of climatic time scales from 10 to 10 000 years can be caused by the stochastic forcing. For a much longer period, Saltzman and Maasch (1991) developed a low-order global model to study the climate change during the late Cenozoic (last 5 million years). However, due to a lack of understanding of the internal dynamics in each of the climate components and of the coupling processes among different climate components, as well as to a lack of computing resources, a real climate model, which includes interactive coupling among different climate components, is still not available at present for the study of the long-term (such as century time scale) natural variability. Furthermore, none of the above studies, except that of Lemke (1977), was aimed specifically at the natural variability over century and millennium time scales. Our study evaluates the natural variability of decadal and century time scales by means of a simple deterministic model.

Obviously, the Lorenz model itself is too simple to be used directly for the study of long-term variability of the earth's climate system. However, the results based on the Lorenz model may be qualitatively correct for different nonlinear systems (including the climate system), and our results lead to our speculation that the seasonal cycle or other prominent periodic variations (such as the 22-year sunspot cycle) may lead to long-term variability due to the internal nonlinear dynamics (especially nonlinear coupling processes) in the earth's climate system. A positive (but indirect) support of this speculation is found in Winograd et al. (1992), where, based on new observational evidence, they found that ice ages may not be caused predominantly by the earth's orbital cycle (i.e., the Milankovitch theory) but by the natural internal variabilities of the climate system. Therefore, with respect to the study of the cause for the natural variability of the climate system, the main value of our study is not the quantitative results themselves but its use in indicating a direction for future research when a real climate model and more powerful computers are available.

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