Atmospheric Parameterization of Evaporation from Non-Plant-covered Surfaces

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ABSTRACT

A new atmospheric parameterization formulation of evaporation from a non-plant-covered surface was derived by combining the previous two types of widely used formulations, the so-called "α method" and the "β method." The study indicates that the "α method" cannot provide a reasonable estimation of evaporation from bare soil, while the "β method" does provide reasonable predictions during the daytime. However, the evaporation rate differences, estimated by the β method from that by the new parameterization, is evident at night. The deviation rises while increasing the atmospheric stability and reducing the soil wetness. The impact of atmospheric stability, soil wetness at the top or inside the soil, and soil type on evaporation rate is also discussed in detail.

1. Introduction

The earth's surface is covered by water (oceans, lakes, rivers, etc.) and different types of soil, which in turn is partly covered by vegetation with the remainder being bare. The soil has different volumetric soil water contents, as a temporary or long-term feature caused by irrigation and heterogeneously distributed precipitation, and by topographical difference. Properly estimating the dependence of evaporation from the ground surface on soil water content is important in expressing the atmosphere and surface interaction and is required in numerical weather and climate models ranging from the global scale to the microscale. Charney et al. (1977), and Shukla and Mintz (1982) indicated that the results from climate model experiments were very sensitive to the soil parameterization. Smith et al. (1986) have shown the importance of near-surface moisture content in the energy budget in a desert area. Three methods of parameterization of evaporation rate $E_0$ from the surface have been used as summarized in Table 1 and listed as follows:

\[
E_0 = \rho_d \frac{q^\text{sat}(T_s) - q_a}{r_a}, \tag{1}
\]

\[
E_0 = \rho_d \beta \frac{q^\text{sat}(T_s) - q_a}{r_a}, \tag{2}
\]

and the combined form of Eqs. (1) and (2):

\[
E_0 = \rho_d \beta \frac{q^\text{sat}(T_s) - q_a}{r_a}, \tag{3}
\]

where $T_s$ is the temperature at the ground surface, $q^\text{sat}$ is the specific humidity at the saturation condition, $q_a$ is the specific humidity of the air, and $\rho_d$ is the air density. Variable $r_a$ is a resistance to water vapor diffusion in air, defined as

\[
r_a = \int_{Z_{0a}}^{Z_a} \frac{1}{K_e} dZ + \frac{0.0962}{k_0 \mu_a} \left( \frac{Z_{0a} \mu_a}{\nu} \right)^{0.45},
\]

in which $K_e$ is the turbulent exchange coefficient at the height $Z$; $Z_{0a}$ and $Z_a$ are aerodynamic roughness for water vapor (Verma 1989) and reference height within the surface flux layer, respectively; $k_0$ is the von Kármán constant; $\nu$ is the kinematic viscosity of air; and $\mu_a$, the friction velocity. The first and second terms on the right-hand side of this equation relate, respectively, to turbulent and molecular exchange processes in the turbulent layer above $Z_{0a}$ and in the laminar layer below. A detailed derivation of this equation is provided in the Appendix.

Nappo (1975) examined the two methods expressed by Eqs. (1) and (2) with the $\alpha$ and $\beta$ expressions as follows:

\[
\alpha = h_s = \exp \left( \frac{\psi g}{RT} \right) \tag{4}
\]

and

\[
\beta = m = \frac{\theta}{h^{\text{sat}}}, \tag{5}
\]

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Table 1. Three methods of atmospheric parameterization of evaporation rate from soil, $E_0$, used in papers.

<table>
<thead>
<tr>
<th>Method</th>
<th>Formula</th>
<th>Reference</th>
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<tbody>
<tr>
<td>$\alpha = \exp \left[ -\frac{\rho a}{RT_s (X_p m_a)^N} \right]$</td>
<td>$\beta = 1.0 - \exp(-56.6 X_p m_a)$</td>
<td>Davies and Allen (1973)</td>
</tr>
<tr>
<td>$\alpha = \frac{\rho a}{RT_s}$</td>
<td>$\beta = m_a$</td>
<td>Nappo (1975) and Segal et al. (1990)</td>
</tr>
<tr>
<td>$\alpha = \min (1, \frac{1.8 X_p m_a}{1.3 m_a})$</td>
<td>$\beta = \min \left(1, \frac{1}{3} m_a\right)$</td>
<td>Deardoff (1978)</td>
</tr>
<tr>
<td>$\alpha = \min (1, \frac{1.8 X_p m_a}{0.3 + X_p m_a})$</td>
<td>$\beta = \left(1 + \frac{r_{sw}}{r_a}\right)^{-1}$</td>
<td>Camillo and Gurney (1986)</td>
</tr>
<tr>
<td>$\alpha = \min (1, \frac{1.8 X_p m_a}{0.4 + 0.7 X_p m_a})$</td>
<td>$r_{sw} = 3.5 m_a^{2.3} + 33.5$</td>
<td>Sun (1982)</td>
</tr>
<tr>
<td>$\alpha = \left{ \begin{array}{ll} 0.5 \left[ 1 - \cos \left( \frac{\pi m_a}{m_g} \right) \right] &amp; , \quad m_g &lt; m_a \ 1 &amp; , \quad \text{otherwise} \end{array} \right.$</td>
<td>$r_{sw} = 3.8113 \times 10^6 \times \exp(-13.515 m_a / m_f)$</td>
<td>Plasser (1986)</td>
</tr>
<tr>
<td>$\alpha = \frac{F_0}{D_r}$</td>
<td>$\beta = a + \frac{1 - a}{1 + \exp(b X_s (m_a - m_g))}$</td>
<td></td>
</tr>
</tbody>
</table>

where $\psi$ is the soil water potential; $g$ is the gravity acceleration; $R$, the gas constant for water vapor; $\theta$, the volumetric water content of the soil; $\theta_{sat}$, the value of $\theta$ at the saturation condition (i.e., the volumetric fraction of soil pores, $X_p$, are completely filled with water); $h$, is the relative humidity in the pores; and $m$ is soil moisture availability.

As indicated by Lee and Pielke (1992), the value of $h$, computed using Philip's (1957) formulation [Eq. (4)] will be overestimated when the soil moisture availability drops to around the soil wilting point. Therefore the evaporation from soils under these conditions, estimated using Eq. (1) with the $\alpha$ value computed by Eq. (4), will also be overestimated. The disadvantage related to Eq. (2) is that the negative surface evaporation rate, which occurs over areas such as a desert area next to the ocean or a snowmelt area (Wang and Mitsuta 1991), cannot be expressed properly by Eq. (2). The negative surface evaporation rate can occur more easily using Eq. (1) than Eq. (2) (Nappo 1975). Mahfouf and Noilhan (1991) indicated after comparing various formulations of evaporation from bare soil using observational data that Eqs. (1) and (2) can provide comparable results during the daytime, while significant differences are exhibited at night.

Another disadvantage concerning evaporation rate from a bare soil surface using Eqs. (1) or (2) is that the value of $E_0$ is independent of $X_p$ (which is 0.395 for sandy soil and 0.863 for peat soil!). On the other hand, when the values of $m$, $T_s$, $q_a$, and $r_a$ are kept the same, $E_0$ from sand and from peat are equal. However, the saturated water content contained in peat soil pores is over two times as large as that in sandy soil. The values of $E_0$, predicted by Eqs. (1) and (2) cannot be distinguished between ocean and saturated soil if we assume $T_s$, $q_a$, and $r_a$ keep the same values for both cases.

It is discussed in this paper that Eq. (3) has the ability to overcome this disadvantage. Furthermore, many authors (such as Barton 1979; Yasuda and Toya 1981; Camillo and Gurney 1986; Dorman and Sellers 1989; Avissar and Mahrer 1988; and Kondo et al. 1990) have indicated the dependence of $E_0$ on $X_p$. Some of them (such as Kondo et al. 1990; Avissar and Mahrer 1988; and Ritchie 1972) also indicated that the evaporation rate from soil depends upon soil type and soil hydraulic
properties. However, the impact of \( \chi_p \) on \( E_0 \) in some formulations presented by Sasamori (1970), Davies and Allen (1973), Barton (1979), and Yasuda and Toya (1981) (see Table 1) shows that \( E_0 \) increases as \( \chi_p \) increases. In contrast, the dependence of \( E_0 \) on \( \chi_p \) in other formulations presented by Avisar and Mahrer (1988), Dorman and Sellers (1989), and Kondo et al. (1990) is just the opposite. These issues motivate us to reexamine the parameterization of evaporation from soil.

The purpose of the current study is to propose a new evaporation formulation from a bare ground surface, which can overcome the disadvantages indicated previously and is able to explain why the \( \beta \) method can provide reasonable results during the daytime but exhibits significant differences at night.

2. Parameterization formulation of evaporation over bare soil

Soil consists of minerals, organic matter (which forms soil particles), water (which fills the soil pores by capillary force), and air located within the remainder of the soil pores. The vapor flux rising from the soil surface into the air directly above relates to two processes: the evaporation from the water surface \( (E_{0w}) \) contributed by the volumetric ground surface water content, \( \theta_s \), associated with a very thin upper soil layer \( \Delta Z_0 \); and vapor diffusion from the remainder of the pores \( (\chi_p - \theta_s) \) into the overlying air, \( E_{0v} \). The evaporation rate from \( \theta_s \) is formulated as

\[
E_{0w} = \rho_d \frac{q_{st}(T_s) - q_a}{r_a},
\]

where \( T_s \) is the soil surface temperature associated with a very thin upper layer, \( \Delta Z_0 \), and computed approximately from a surface energy balance equation.

The vapor flux by a diffusion process from a pore’s surface into the air is expressed as

\[
E_{0v} = \rho_d \frac{q_s - q_a}{r_a},
\]

where \( q_s \) is the specific humidity at the soil surface. Note that the same resistance in Eqs. (6) and (7) was assumed, since the resistance is controlled by the same atmospheric characteristics with scales that are much larger than the scale of soil pores, and \( q_a \) is the value at the blending height (Wieringa 1986). Furthermore, we assumed that the mixing between \( E_{0v} \) and \( E_{0w} \) may be unimportant below the blending height.

Within the soil pores, vapor flux transported from a thin layer of depth \( \Delta Z_1 \) upward by molecular diffusion is computed by

\[
E_{1v} = \rho_d \frac{h_s(\theta_1)q_{st}(T_{st}) - q_s}{r_D},
\]
and

\[
\alpha = \frac{1}{\beta} \left[ \theta_g + \frac{x_p(1) - \theta_1}{x_p(1) - \theta_1} \frac{r_a}{h_s(\theta_1)} \frac{\frac{\partial Q^s}{\partial \theta}}{\frac{\partial Q^s}{\partial \theta}} \right].
\]

(15)

3. Results and discussion

The previous formulations of evaporation from the soil surface, expressed by Eqs. (1) and (2), are called the “\( \alpha \) method” and “\( \beta \) method,” respectively. The present parameterization formulation, given by Eq. (13), is a combination form which we refer to as the “\( \alpha \) and \( \beta \) method.” A similar version with different expressions of \( \alpha \) and \( \beta \) has been presented in Table 1.

\textbf{a. The impact of ratio} \( r_a/r_D \) \textbf{and soil moisture availability on} \( \beta \)

Equation (14) divided by \( x_p \) yields

\[
\beta_* = \frac{\beta}{x_p} = 1 - \frac{1 - m_g}{1 + \frac{x_p(1) - m(1)}{x_p(g)}} \frac{r_a}{r_D},
\]

(16)

where \( m \) is the soil moisture availability, defined by Eq. (5).

A representative value of \( r_D \) is 400 s m\(^{-1}\). The value of \( r_a \) is dependent on \( Z_{0,a} \), ambient wind speed, and atmospheric thermal stability, ranging from 10\(^2\) s m\(^{-1}\) under very stable stratified atmospheric conditions with a very smooth ground surface, to about 10\(^0\) s m\(^{-1}\) in a very rough surface under unstable conditions. Therefore the ratio \( r_a/r_D \) changes its values from 10\(^0\) to 10\(^{-2}\) under very unstable situations. For a given soil type and soil wetness condition, Eq. (16) describes that (a) when the ratio of \( r_a/r_D \) increases (i.e., when the atmospheric thermal stability tends to be more stable or less unstable) \( \beta_* \) increases; (b) for given soil types and a ratio of \( r_a/r_D \), the value of \( \beta_* \) decreases as the drying process proceeds in initially wet soil, because of the reduction of the value \( m_g \) and the increment of \([1 - m(1)]/(1 - mg)\); (c) generally \( \beta_* < 1 \), but \( \beta_* \to 1 \) when \( m_g \to 1 \); and (d) generally, \( \beta_* < m_g \), but \( \beta_* \to m_g \) when \( r_a/r_D \to 0 \).

Figure 1 presents the impact of \( r_a/r_D \) on \( \beta_* \) for three different values of \( m_g \): 0.05 (profile A), 0.25 (profile B), and 0.5 (profile C), respectively, under the assumption that soil type and soil moisture availability were uniformly distributed (this assumption is used in all figures presented below). It illustrates that the impact of \( r_a/r_D \) on \( \beta_* \) intensifies as soil moisture availability decreases. When soil moisture is small (\( m = 0.05 \), for example), the value of \( \beta_* \) expands its value by an order of magnitude when \( r_a/r_D \) increases from 0.01 to 1.0 (see profile A). The relatively abrupt alteration of \( \beta_* \) with \( r_a/r_D \) occurs with relatively smaller values of \( r_a/r_D \).

The dependence of \( \beta_* \) on \( m_g \) and \( m(1) \) can be explored by differentiating Eq. (16) with respect to \( m_g \) and \( m(1) \), respectively,

\[
\frac{d \beta_*}{dm_g} = 1 - \left[ \frac{r_a}{r_D} \frac{1 - m(1)}{x_p(g)} \right]^2,
\]

(17)

and

\[
\frac{d \beta_*}{dm(1)} = - \frac{r_a}{r_D} \frac{x_p(1)}{x_p(g)} \left[ 1 + \frac{x_p(1)}{x_p(g)} \frac{r_a}{r_D} \frac{1 - m(1)}{1 - m_g} \right]^2.
\]

(18)

Equation (17) indicates that (a) the slope \( d \beta_*/dm_g \) is always positive, since the second term on the right-hand side of Eq. (17) is less than one; (b) the slope increases as the value of \( r_a/r_D \) decreases; and (c) when \( r_a/r_D \ll 1 \), \( d \beta_*/dm_g \to 1 \). Equation (18) describes that (a) since the value on the right-hand side of Eq. (18) is negative for any values of \( r_a/r_D \) and \( 1 - m(1)/(1 - m_g) \), the value of \( \beta_* \) always reduces as the soil moisture availability increases inside the soil layer; (b) the slope, \( -d \beta_*/dm(1) \), is generally less than one; and (c) the slope, \( -d \beta_*/dm(1) \), will linearly increase with \( r_a/r_D \) under the condition \( r_a/r_D \ll 1 \).

Figure 2, computed from Eq. (16) with \( r_a/r_D = 1.0 \) (profile A), 0.1 (profile B), and 0.01 (profile C), respectively, and the other conditions as used in Fig. 1,
illustrates that the value of $\beta_e$ increases linearly with the value of $m_e$. When the value of $r_a/r_D$ is small (profile C), $\beta_e$ is approximately equal to $m_e$. The deviation of $\beta_e$ from $m_e$ ($\beta_e > m_e$) increases as the atmospheric stability increases for a fixed value of $m_e$, and decreases as the soil moisture availability increases for a fixed value of $r_a/r_D$.

The above discussion implies that the value of $\beta_e$ is dependent on both the atmospheric stability near the surface and the soil moisture availability.

b. The dependence of $\alpha$ on $r_a/r_D$ and soil moisture availability

Combining Eqs. (15) and (16), to eliminate $\beta$ results in

$$\alpha = \frac{1}{\beta_e} \left\{ m_e + \frac{\left[ 1 - m(t) \right] x_p(1) x_p(g)}{1 + x_p(1) 1 - m(t) r_a x_p(g) 1 - m_e r_D} \right\} \times \frac{r_a q_{out}[r_a(t)]}{r_D q_{out}[r_D]} h_s[m(t)].$$

The value of $h_s$ as dependent on $m$ is computed using the formulation given by Jacquemin and Noilhan (1990):

$$h_s(m) = \begin{cases} 0.5 \left[ 1 - \cos \left( \pi \frac{m}{m_e} \right) \right], & \text{if } m \leq m_e \\ 1, & \text{otherwise,} \end{cases}$$

where the subscript $f_c$ stands for field capacity of soil. The values of $m_{fc}$ for different soil types can be found from Lee and Pielke (1992) and Cosby et al. (1984).

Figure 3, computed based on Eq. (19), presents the dependence of $\alpha$ on $r_a/r_D$ for three cases of soil moisture availability: $m_e = 0.05$ (profile A), 0.25 (profile B), and 0.5 (profile C) under the conditions that the soil type, soil moisture availability, and soil temperature are distributed uniformly, with $m_{fc} = 0.366$ for loamy sand soil (Lee and Pielke 1992).

For $m_e = 0.05$, the value of $\alpha$ decreases nonlinearly from about 0.85 to about 0.13 when $r_a/r_D$ increases from 0.01 to 1.0. When the soil moisture availability increases, the dependence of the value of $\alpha$ on $r_a/r_D$ is weakened. When the soil moisture availability is greater than $m_{fc}$, the value of $\alpha$ is equal to one, which is independent of $r_a/r_D$. The value of $\alpha = 1$ when $r_a/r_D \rightarrow 0$ for any value of $m_e$.

Figure 4 illustrates the impact of $m_e$ on $\alpha$ for three cases of $r_a/r_D = 1.0$ (profile A), 0.1 (profile B), and 0.01 (profile C) with the same conditions as indicated in Fig. 3. The profile D presents the dependence of $h_s$ on $m_e$ computed based on Eq. (20). It shows that (a) for any values of $r_a/r_D$, increasing the soil moisture availability results in the increase of $\alpha$ when $m < m_{fc}$; when $m > m_{fc}$, we have $\alpha = 1$; (b) the gradient of $\alpha$ with respect to $m$ is sharpened as the value of $r_a/r_D$ decreases (or as the atmospheric instability increases); and (c) comparing profile D with profiles A, B, C indicates that $\alpha > h_s$, when $m_e < m_{fc}$. The deviation of $\alpha$ from $h_s$ increases as the value of $r_a/r_D$ is decreased, which means that when the atmospheric stability tends to be more stable, the $\alpha$ profile will approach the $h_s$ profile.

![Figure 3](image3.png)

Fig. 3. The same as in Fig. 1 except for $\alpha$ instead of $\beta_e$.
It suggests that when the soil moisture availability is greater than the soil field capacity or \( r_a/r_d \to 0, \alpha = 1 \), and Eq. (13) becomes the \( \beta \) method.

c. The impact of \( r_a/r_d \), and \( m \) on \( E_0 \)

Equation (13) can be expressed as

\[
E_0 = \frac{\rho_d}{r_a} \beta (\alpha - hh_c)q_{\text{sat}}^{\text{soil}},
\]

where \( h \) is the atmospheric relative humidity, defined as

\[
h = \frac{q_a}{q_{\text{sat}}}
\]

and \( hh_c = \frac{q_{\text{sat}}}{q_{\text{sat}}^{\text{soil}}} \).

The value of \( hh_c \) is dependent on the air temperature deviation from the surface temperature (i.e., on the atmospheric thermal stability).

Equation (21) indicates that (a) for fixed values of \( h \), \( hh_c \), \( \alpha \), \( \beta \), and \( r_a \), higher temperature corresponds to a higher value of \( q_{\text{sat}} \), which results in a higher value of \( E_0 \); (b) there will be no water vapor flux when

\[
hh_c = \alpha;
\]

and (c) during the daytime, the atmospheric thermal stability is in an unstable condition, \( hh_c < 1 \). The value of \( hh_c \) decreases as atmospheric instability increases. Generally, \( \alpha > hh_c \), holds, so \( E_0 > 0 \). The value of \( E_0 \) increases as the atmospheric relative humidity \( h \) decreases, and/or the atmospheric instability increases (i.e., decreasing the value of \( hh_c \)), and/or the soil moisture availability increases, which increases both values of \( \alpha \) and \( h \), as discussed in sections 3a and 3b. Note that the value of \( \alpha \) decreases as the value of \( r_a/r_d \) increases (increasing the stability) and/or the soil moisture availability decreases. Therefore, under a suitable condition (dry soil with overlying humid air), even in the daytime, a negative vapor flux (i.e., condensation) will be able to occur; and (d) at night, a surface radiative inversion is formed, which results in increasing both values, \( r_a \) and \( h_c (h_c > 1) \), and the value of \( h \) is enhanced because the temperature drops. As compared to the daytime situation, \( E_0 < 0 \) at night is able to occur more easily. Turbulent intensity characterized by \( r_a^{-1} \) at night is usually one order of magnitude smaller than during the day. Therefore, the value of \( E_0 \) at night is commonly smaller as compared to that during the day.

Figure 5 presents an example of the impact of \( r_a/r_d \) on \( E_0 \) in three cases, \( m = 0.05 \) (profile A), 0.25 (profile B), and 0.5 (profile C), under conditions \( r_d = 400 \) s m\(^{-1} \), \( T_e = 26^\circ \text{C} \), and \( T_a = 22^\circ \text{C} \), \( h = 60\% \), during the day (Fig. 5a) and \( T_e = 22^\circ \text{C} \), \( T_a = 26^\circ \text{C} \), \( h = 71\% \) at night, for a loamy sand soil uniformly distributed, where \( E_0 = E_0/r_aX_p \). Figure 5a illustrates the dependence of \( E_0 \) (m s\(^{-1} \)) on \( r_a/r_d \) during the day. It illustrates that the values of \( E_0 \) for the three cases decrease by one order of magnitude when the value of \( r_a/r_d \) increases from 0.01 to 0.1. A sharper decrease occurs when \( r_a/r_d > 0.02 \). A negative value of \( E_0 \) during the day is computed for a very dry soil with \( m = 0.05 \) and \( r_a/r_d > 0.076 \). However, the value of \( E_0 \) is very small, since the turbulence is weak. Therefore, \( E_0 > 0 \) is common during the daytime.

At night (Fig. 5b), both \( E_0 > 0 \) and \( E_0 < 0 \) are computed with the given conditions indicated above. The dependence of the absolute values of \( E_0 \) on \( r_a/r_d \) at night exhibits the same rule as that during the day. However, the positive and negative water vapor fluxes at night are much smaller than those during the day. When the soil is dry (\( m = 0.05 \)) with a high air relative humidity (\( h = 71\% \)) and inversion intensity is 4°C, condensation occurs (profile A). The amount of condensation water increases as the turbulent intensity increases (i.e., decreasing the value of \( r_a/r_d \)). When the soil moisture availability \( m = 0.5 \) is above the soil field capacity, under the same given conditions, the values of \( E_0 \) are positive. When \( m = 0.25 \), the value of \( E_0 \) decreases from positive to negative when the value of \( r_a/r_d \) increases from 0.12 to 1.0. The transition point is at around \( r_a/r_d = 0.4 \) for the given condition, which will decrease as the soil moisture availability decreases or both values of the air relative humidity and the inversion intensity increase.

Figure 6a depicts the impact of soil moisture availability on \( E_0 \) for three values of \( r_a = 4 \) s m\(^{-1} \) (profile D), \( r_a = 40 \) s m\(^{-1} \) (profile C) during the day, \( r_a = 40 \)
FIG. 5. The impact of $r_o/r_0$ on $E_a$ (m s$^{-1}$), with $r_0 = 400$ m s$^{-1}$, $h = 1000$ mb, for $m_g = 0.05$ (profile A), 0.25 (B) and 0.5 (C), respectively, under the conditions, loamy sand soil with moisture availability uniformly distributed and $T_a = 26^\circ$C, $T_e = 22^\circ$C, $h = 60\%$ for (a); and $T_a = 22^\circ$C, $T_e = 26^\circ$C, $h = 71\%$ for (b).

s m$^{-1}$ (profile B), and $r_o = 400$ s m$^{-1}$ (profile A) at night under the same conditions as indicated in Fig. 5. Figure 6b is a higher resolution portion of Fig. 6a. It illustrates that during the day, except under very dry soil conditions ($m_g \leq 0.05$), the value of $E_a$ is positive, and it increases rapidly with increasing value of $m_g$. The value of $E_a$ increases from $1.17 \times 10^{-4}$ m s$^{-1}$ at $m_g = 0.05$ to $2.6 \times 10^{-3}$ m s$^{-1}$ at $m_g = 0.95$ in the case of $r_o = 4$ s m$^{-1}$. Negative vapor flux can occur during the day only in a very dry soil condition as illustrated in Fig. 6b. The value of $m_{tg}$, at which the vapor flux is changed from negative to positive, increases as the atmospheric turbulent intensity decreases (the transition value of $m_{tg} \approx 0.007$ when $r_o \rightarrow 4$ s m$^{-1}$ and $m_g = 0.06$, while $r_o = 40$ s m$^{-1}$ is computed as shown by profiles C and D in Fig. 6b). The transition

FIG. 6. (a) The dependence of $E_a$ (m s$^{-1}$) on soil moisture availability for $r_o/r_0 = 0.01$ (profile D), 0.1 (profile C) during the day, $r_o/r_0 = 0.1$ (profile B), and 1.0 (profile A) at night, respectively, with the other conditions the same as used in Fig. 5. (b) A magnification of the lower-left portion of (a).
point of \( m_g \) also increases as the humidity of the overlying air increases. The gradient of \( E_a \) with respect to \( m_g \) is much sharper as compared to the change of \( E_a \) with \( r_a/r_d \). The reason is that when the value of \( m_g \) increases, both values of \( \alpha \) and \( \beta_a \) increase rapidly, as shown by Figs. 2 and 4. However, when the value of \( r_a/r_d \) increases, the value of \( \beta_a \) increases, but the value of \( \alpha \) decreases, as shown by Figs. 1 and 3. At night, the value of \( E_a \) changes its sign at \( m_g \approx 0.225 \) for \( r_a = 40 \) s m\(^{-1}\) and at \( m_g \approx 0.275 \) for \( r_a = 400 \) s m\(^{-1}\) for the given conditions, which suggests that for this situation, weak turbulence favors the occurrence of condensation. The amount of condensation water decreases, however, as the soil wetness increases and/or the turbulent intensity decreases. Figure 6 also illustrates that at night the values of \( E_a \) are much smaller as compared to the values during the day. The dependence of \( E_a \) on \( m_g \) is linear, but on \( r_a/r_d \) is nonlinear as illustrated by Figs. (5) and (6).

d. Discussion

Combining Eqs. (13), (16), and (23) yields

\[
E_o = \rho_d \chi_p(g) \beta_a \frac{[\alpha q(T_g) - q_a]}{r_a} \tag{25a}
\]

\[
E_o = \rho_d \chi_p(g) \beta_a \frac{(\alpha - \delta h) q(T_g)}{r_a} \tag{25b}
\]

Equation (25) indicates that for fixed values of \( \beta_a \), \( \alpha \), \( T_g \), \( q_a \), and \( r_a \) (i.e., for fixed values of \( T_g \), \( \Delta T \), \( q_a \), and \( r_a \)), the water vapor flux from (or to) the ground surface is linearly proportional to the soil porosity volumetric fraction under the condition that the soil type is unchanged within a shallow soil layer, because both \( \alpha \) and \( \beta_a \) in this condition are independent of \( \chi_p \). When \( \chi_p(g) \neq \chi_p(1) \), \( E_o \) nonlinearly depends on \( \chi_p(g) \). This result is different from the previous evaporation formula given by Nappo (1975), Deardorff (1978), Jacquemin and Noilhan (1990), and others (see Table 1 for details). In these formulas of evaporation, the evaporation rate is the same for peat soil (\( \chi_p = 0.863 \)) and sand soil (\( \chi_p = 0.395 \)), which is unreasonable. The dependence of \( E_o \) on \( \chi_p \) given by the present parameterization qualitatively agrees with that given by Sasamori (1970), Davies and Allen (1973), Barton (1979), and Yasuda and Toya (1981) (see Table 1 for details).

Since the new formula contains the dependence of \( E_o \) on \( \chi_p(g) \), Eq. (25) can be extended to estimate the evaporation rate from free open water bodies (such as rivers, lakes, and oceans). These free open water bodies can be imagined as a special soil with \( \chi_p(g) = m = 1 \). Under this condition, \( \beta_a \) and \( \alpha \), computed from Eqs. (16) and (19), are equal to 1. Therefore, Eq. (25) used to estimate the evaporation rate from a free open water body simplifies to

\[
E_o = E_w = \frac{\rho_d}{r_a} [q_{out}(T_g) - q_a]. \tag{26}
\]

The above discussion suggests that for given values of \( T_g \), \( q_a \), and \( r_a \), the evaporation rate from an open water body is larger than that from any type of soil even under saturated soil water condition. Penman (1948) provided experimental data measured from two cylinders filled with water and a sandy loam, respectively, which supports this opinion. He indicated that the evaporation rate from wet bare soil with an adequate supply of water is obtained as a fraction of that from open water. However, the surface roughness length over soil is commonly much larger than that over free open water bodies, and \( T_g \) for soil is higher in daytime than that for free open water under the same overlying environmental conditions. That is why the evaporation rate measured over soil can be larger than that from an adjacent water body during the day.

It is necessary to clarify that Eq. (25) describes the water vapor flux at the surface, not the evaporation rate from the surface. These are equal for a soil surface with \( m_g = 1.0 \) or over an open water body. In the case that \( m_g < 1 \), however, the value of \( E_o \) consists of two portions; one is evaporated from \( \beta_a \) which consumes latent heat and will take part in the heat balance equation at the surface. The remainder of \( E_o \) is contributed by vapor diffusion through soil pores as the vapor is evaporated at different depths of the soil and will, therefore, take part in the heat balance equation at different depths of the soil.

From the discussion in sections 3a and 3b, except for \( m_g = 1 \) the values of \( \beta_a \) are always less than one. Therefore the \( \alpha \) method can never be reached. However, on some occasions, \( \alpha \) is equal to or approaches one and the \( \beta \) method is a good approximation to Eq. (25). These occasions include (a) when the soil moisture availability is about equal to or greater than the soil field capacity (see Fig. 4 for details). In this soil wetness condition, the value of \( \beta_a \) is about equal to the value of \( m_g \), when \( r_a/r_d \ll 0.1 \), as shown by profiles B and C in Fig. 2. When the value of \( r_a/r_d \) increases, the deviation of \( \beta_a \) from \( m_g \) also increases, as illustrated by profile A in Fig. 2; \( \beta_a/m_g = 1.9 \) was computed under the conditions \( r_a/r_d = 1.0 \), \( m_g = m_{fc} \), as shown by Fig. 2; and (b) from Eqs. (16) and (19), when \( r_a/r_d \ll 1 \), \( \beta_a \approx m_g \) and \( \alpha \approx 1 \) can be derived. These features can also be seen from Figs. 1 and 3. Figures 1 and 3 illustrate that when \( r_a/r_d \ll 0.035 \), the values of \( \alpha \) and \( \beta_a \) are as follows: \( \alpha = 1.0 \), \( \beta_a = 0.505 \pm 0.517 \) for \( m_g = 0.5 \); \( \alpha = 0.97 \pm 0.993 \), \( \beta_a = 0.257 \pm 0.276 \) for \( m_g = 0.25 \); and \( \alpha = 0.622 \pm 0.849 \), \( \beta_a = 0.059 \pm 0.083 \) for \( m_g = 0.05 \). This discussion indicates that for the conditions of \( r_a/r_d \ll 0.035 \), \( m_g > 0.25 \) (about 70% of \( m_{fc} \))
the values of $\alpha$ and $\beta_*$ can be expressed as $\alpha = 1$ and $\beta_* \approx m_\alpha$ [i.e., Eq. (5) holds]. That is to say that under these conditions, Eq. (25) is simplified to a $\beta$ method. For a representative value of $r_a = 400$ s m$^{-1}$, $r_a/r_D < 0.035$ implies $u_a > 0.2$ m s$^{-1}$ in neutral atmospheric condition when we set $r_a \approx k u_a$. A value of $u_a > 0.2$ m s$^{-1}$ is common in daytime unstable boundary conditions.

In order to further estimate the accuracy of the computed water vapor flux using the $\beta$ method, the following equation can be derived from Eq. (25):

$$F_\beta = \frac{E_0 - E_{0b}}{E_0} = \frac{\alpha - 1}{\alpha - hh_c}$$

where $E_0$ is expressed by Eq. (25), $E_{0b}$ is expressed by Eq. (25) with $\alpha = 1$. Then, the accuracy of the $\beta$ method is determined by the value of $F_\beta$ as follows: for $F_\beta < 0$, the $\beta$ method provides an overestimate, for $F_\beta > 0$, the $\beta$ method is an underestimate, while for $F_\beta = 0$, the $\beta$ method is accurate.

From the discussion above, the values of $\alpha$ computed based on Eq. (19) were from 0.9 to 0.98 with $m_\alpha = 0.2$ when $r_a/r_D$ changed its values from 0.1 to 0.01 during the day (i.e., $\alpha \rightarrow 1$ when $r_a/r_D \rightarrow 0$), and $\alpha = 1$ for any value of $r_a/r_D$ when $m_\alpha \approx m_{f_c}$. However, $\alpha < 1$ when the value of $r_a/r_D$ increases under the condition of $m_\alpha < m_{f_c}$. It suggests that during the daytime for situations of low synoptic wind and cloudy skies, the $\beta$ method is a good approximation of Eq. (25). It will fail when the atmospheric turbulent intensity decreases and the soil is in a very dry condition during the day. For this condition, when $hh_c < \alpha < 1$, $F_\beta < 0$, and the $\beta$ method overestimates the evaporation from the soil, alternatively $F_\beta > 0$ when $\alpha < hh_c < 1$, so that the $\beta$ method underestimates the evaporation from the soil, but it occurs only rarely during the day because of the need for small values of $h$ and $h_c$.

At night, the $\beta$ method is accurate only under the condition of $m_\alpha > m_{f_c}$ and $\alpha \neq hh_c$. For the remainder of the conditions, however, $\alpha < 1$ which is already shown by Fig. 3. The deviation of $\alpha$ from 1 for a given $m_\alpha < m_{f_c}$ increases as $r_a/r_D$ increases, suggesting $|F_\beta|$ is larger at night as compared with that during the day for the same conditions. On the other hand, the values of $h_c$ range from 1 to 1.7 as $\Delta T$ changes from 0 to 10 K, computed based on Eq. (23). The decreased temperature at night makes $h$ increase after sunset. Therefore situations of $\alpha < hh_c < 1$ and $hh_c < \alpha < 1$ are equally likely. Further, $F_\beta > 0$, which occurs in favorable conditions of relatively dry soil or large values of $r_a/r_D$ (both result in a smaller value of $\alpha$), or strength inversions (which yields a larger value of $h_c$), and increasing relative humidity.

The results can be used to qualitatively explain Mahfouf and Noilhan's (1991) conclusions. They indicated after discussing the $\beta$ method based on in situ data collected over a silty clay loam bare soil, with soil surface moisture availability decreasing from field capacity to dry conditions that, during the daytime, it provided comparable results as contrasted to observed data while significant differences were exhibited at night. They also indicated, based on comparing the accumulated evaporation values that, computed by the $\beta$ method (e.g., tests 2 and 3 in their paper) and deduced from both observed water budget and aerodynamic measurements, the $\beta$ method (tests 2 and 3) gives quite similar results and clearly overestimates the integrated evaporation value as a consequence of the assumption $h = \alpha = 1$. Since the evaporation rate during the daytime is one order of magnitude larger than that at night as mentioned above, we can anticipate that the accumulated evaporation rate estimated using the $\beta$ method will be overestimated based on Eq. (27).

The above procedure can also be applied to estimate the accuracy of the $\alpha$ method, where the $\alpha$ method is the formulation of Eq. (25) with $\beta_* = 1$. Then we have

$$F_\alpha = \frac{E_0 - E_{0a}}{E_0} = 1 - \frac{1}{\beta_*},$$

with $E_{0a}$ the evaporation rate computed with the $\alpha$ method.

When $F_\alpha > 0$, the $\alpha$ method underestimates the evaporation from a bare soil, while with $F_\alpha < 0$, the $\alpha$ method overestimates the evaporation. Figures 1 and 2 illustrate that $\beta_* < 1.0$ at any value of $r_a/r_D$ and $m_\alpha$, except $m_\alpha = 1.0$ (where, $\beta_* = 1$). Therefore, Eq. (28) suggests that the $\alpha$ method always overestimates evaporation. The overestimation decreases with increasing values of $r_a/r_D$ and $m_\alpha$ as shown in Figs. 1 and 2.

4. Conclusions

In the present study, a new parameterization formulation of water vapor flux at the interface between air and an underlying non-plant-covered ground surface was derived. The new formulation is a combination of previous $\alpha$ and $\beta$ formulations. The present study also investigated the sensitivity of the impact of $r_a/r_D$ and $m_\alpha$ on $\alpha$, $\beta$, and $E_0$. The major conclusions we obtained from this study follow.

- Based on the results computed from Eq. (16), over unsaturated soil the value of $\beta_*$ was always less than one during the daytime and at night. Therefore, the $\alpha$ method cannot provide a reasonable prediction of evaporation from bare soil; the $\alpha$ method always overestimates the evaporation rate from unsaturated soil.
- The $\beta$ method can provide as good a prediction in estimating the daytime evaporation rate as the one computed by Eq. (25). However, the evaporation rate estimated by the $\beta$ method will deviate significantly from that predicted by Eq. (25) at nighttime. Overestimation or underestimation can occur at night de-
pending on whether \( \alpha \) is greater or smaller than \( hh_c \). The difference increases when the soil is drier.

- The new parameterization formula of evaporation from a bare soil is explicitly dependent on soil type as indicated by soil porosity unlike the \( \alpha \) and \( \beta \) methods. This formula can also be used to estimate the evaporation over open water bodies (such as lakes and oceans) by setting \( \chi_p = 1.0 \).

- The values of \( \alpha \) and \( \beta_e \) are sensitive to the atmospheric stability and soil moisture availability. When the atmospheric stability and/or the soil moisture availability increases, the value of \( \beta_e \) rises. The value of \( \alpha \) is reduced as the atmospheric stability increases or the soil moisture availability is reduced. The influence of \( \alpha \) on \( \beta_e \) intensifies as the soil moisture availability decreases. The influence of soil moisture availability on \( \alpha \) and \( \beta_e \) becomes more important as the availability decreases. The influence of soil moisture availability on \( \alpha \) and \( \beta_e \) becomes more important as the atmospheric instability increases.

- The sign of water vapor flux near the surface depends on the value of \( \alpha - hh_c \). During the day, negative vapor flux can occur only under very dry soil conditions (\( m_2 < 0.06 \) when \( r_a/r_d = 0.1 \) and \( m_2 < 0.007 \) when \( r_a/r_d = 0.01 \) were computed with \( h = 60\% \)). At night, however, negative vapor flux occurs more easily as compared to daytime (\( m_2 < 0.225 \) for \( r_a/r_d = 0.1 \) and \( m_2 < 0.275 \) for \( r_a/r_d = 1 \) were computed with \( h = 71\% \)). The absolute value of \( E_0 \) during the day is an order of magnitude larger than at night since the value of \( r_a \) at night is one order of magnitude larger than that during the day. The absolute value of \( E_0 \) is proportional to \( m_2 \) and decreases nonlinearly with \( r_a/r_d \). The impact of \( m_2 \) on \( E_0 \) is more sensitive as compared to the impact of \( r_a/r_d \), since \( \alpha \) and \( \beta_e \) increase as the value of \( m_2 \) increases; however, as \( r_a/r_d \) increases, \( \beta_e \) increases but \( \alpha \) decreases.

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APPENDIX A

Derivation of \( r_a \)

The expression for the deviation of \( q_t \) at the interface between soil and air from \( q(Z_{0a}) \) as a function of \( u_*, q_*, Z_{0a}, \) and \( \nu \), based on Zilitinkevich (1970) is

\[
q_t - q(Z_{0a}) = -0.0962 \frac{q_*}{k_0} \left( \frac{u_* Z_{0a}}{\nu} \right)^{0.45},
\]

\[
= -0.0962 \frac{u_* q_*}{k_0 u_*} \left( \frac{u_* Z_{0a}}{\nu} \right)^{0.45},
\]

where \( q_* \) is the turbulent scaling parameter for water vapor.

The water vapor fluxes within the laminar layer (0 < \( Z < Z_{0a} \)), \( \rho \Delta q_t - q(Z_{0a})/r \) and in the surface flux layer, \( -\rho \Delta q_t q_* = \rho \Delta q(Z_{0a}) - q_0 \)/\( r_a \) are equal under equilibrium conditions, where

\[
r'_a = \int_{Z_{0a}}^{Z_{0a}} \frac{1}{k_0 u_*} dZ;
\]

so that for \( r' \) and \( E_0 \) we have

\[
r' = \frac{0.0962}{u_*} \left( \frac{Z_{0a} u_*}{\nu} \right)^{0.45},
\]

\[
E_0 = \rho a \frac{q_t - q_0}{r_a},
\]

where \( r_a = r'_a + r' \).

For typical values of \( u_* \approx 10^0 \sim 10^1 \) cm s\(^{-1} \), \( Z_{0a} \) for soils 0.1 \( \sim 1 \) cm, and \( \nu = 0.15 \) cm\(^2\) s\(^{-1} \), \( r' \) we have \( r' \approx 1/k_0 u_* \). In a neutral or stable condition, \( r'_a \approx (c/k_0 u_* \ln(Z_{0a}/Z_{0a})) \), where \( c = K_n/K_0 \), is the ratio of turbulent exchange coefficients for momentum and water vapor.

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