The Derivation of a Terrain-Following Coordinate System for Use in a Hydrostatic Model

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(Manuscript received 7 December 1980, in final form 24 March 1981)

ABSTRACT

This article uses tensor transformation procedures in order to derive a terrain-following coordinate system that is frequently used in a number of regional and mesoscale hydrostatic models. Tensor transformation procedures are used so as to ensure physical invariance of the primitive equations between the Cartesian and terrain-following systems. Among the major conclusions are as follows:

- Applying the chain rule to the hydrostatic equation, before transforming from a Cartesian to a terrain-following coordinate system, yields a different set of equations than if the hydrostatic assumption is applied after the tensor transformation is made. The hydrostatic equations in the two terrain-following representations are the same only when the slope of the terrain in the model is much less than 45°.
- Variations of the metric tensor across a grid volume appear in the set of conservation equations as a result of averaging the equations over a grid volume. Such deviations have always been ignored in existing non-hydrostatic and hydrostatic meteorological models.
- Care must be taken to assure that parameterizations which are a function of distance above the ground be defined in terms of the original Cartesian system, and not the new generalized vertical coordinate \( \sigma \). The profile exchange coefficient \( K(z) \), for example, cannot be defined simply by replacing \( z \) by \( \sigma \).

1. Introduction

The use of a terrain-following coordinate system in meteorological modeling was first introduced by Phillips (1957), and it has since been shown to be an effective mathematical representation. This concept of defining a coordinate surface coincident with the bottom topography permits more efficient use of computer resources, and it simplifies the application of lower boundary conditions. In Phillips' original form, adopted by many models [e.g., the U.S. Weather Service forecast models, Rieck (1979)], pressure is used to define the independent vertical coordinate \( \sigma \), where surface pressure is used as the lower boundary. Haltiner (1971), for example, defines \( \sigma = p/p_s \), where \( p_s \) is the surface pressure while \( p \) is the pressure at any level. For this example \( \sigma = 1 \) corresponds to the ground surface.

In recent years, \( \sigma \) has often been defined as a function of height rather than pressure. This is advantageous because \( p_s \) is a function of time, whereas terrain height is not. The general form of this coordinate system transformation is given as

\[
\sigma = \frac{s - z_G}{s - z_G},
\]

where \( s \) is usually defined as a constant (generally defined as the top of the model)\(^1\) while \( z_G \) is the terrain height. The variable \( z \) is height, while \( \sigma \) is referred to as a generalized vertical coordinate. This form of a terrain-following coordinate has been used in recent years in regional and mesoscale models (e.g., Mahrer and Pielke, 1975; Kasahara, 1974; and others) in which the hydrostatic assumption has been applied.

In developing their equations, however, these investigators have applied the chain rule separately in the vertical and horizontal dimensions (utilizing the hydrostatic relation). Using (1), this results in the transformed hydrostatic equation given as

\[
\frac{\partial \pi}{\partial \sigma} = -\frac{s - z_G}{s} \frac{\theta}{\theta},
\]

where \( \pi = c_s T/\theta \). This is appropriate if the hydrostatic assumption is exactly satisfied. However, the invariance of the physical representation (which must be retained, regardless of the coordinate formulation) is lost if the assumption is not exact.

\(^1\) Mahrer and Pielke (1975, 1978) permitted \( s \) to vary in the denominator, but this complicates the analyses somewhat. Therefore, in this paper, \( s \) is assumed to be a constant. The conclusions from the analyses are not significantly affected by this requirement. See Dutton (1976, p. 251) for the modifications needed when the coordinate transformation is time dependent.
2. The equation of motion

Dutton (1976) demonstrated that the contravariant form of the equation of motion in a generalized coordinate system, derived from the rectangular x-y-z (x') system, can be written as

$$\frac{\partial \vec{u}^i}{\partial t} + \hat{u}^i \frac{\partial \vec{u}^i}{\partial x^j} = -\hat{G}^{ij} \frac{\partial \theta}{\partial x^j} - \frac{\partial \hat{r}^i}{\partial x^j} - 2\hat{e}^{ij} \hat{\Gamma}_j^k \hat{u}^k, \quad (3)$$

where $\vec{u}^i$ is the contravariant component of velocity, $\hat{G}^{ij}$ the contravariant metric tensor, $\hat{r}^i$ represents the independent variable in the new coordinate system, and

$$\hat{e}^{ij} = \epsilon_{ijl} \hat{G}^{-1/2} \hat{u}^l, \quad \hat{\Gamma}_j^k = \frac{\partial \hat{u}_j}{\partial x^k} + \hat{\Gamma}_j^k \hat{u}^k.$$

The term $g^{ij}(\partial \hat{r}^j/\partial x^i)$ is obtained from $\hat{G}^{ij} \partial \rho \partial \hat{r}^i$. The tilda is used to indicate a variable in the transformed coordinate system, while $\epsilon_{ijl}$ is $\epsilon_{ijl}$ in the Cartesian system. The tensor $\epsilon_{ijl}$ is defined as zero if any two of the indices are equal, +1 if an even permutation of the indices occurs, and -1 with an odd permutation. The parameter $G$ is the determinant of the contravariant form of the metric tensor $\hat{G}^{ij}$ while $\hat{\Gamma}_j^k$, called the Christoffel symbol, is given by

$$\hat{\Gamma}_j^k = \frac{\partial \hat{r}^i}{\partial x^m} \frac{\partial \hat{r}^m}{\partial x^k} \frac{\partial \hat{r}^i}{\partial x^j}.$$
\[
\begin{align*}
\mathbf{u}_i & = \begin{cases} 
\frac{\partial \tilde{u}^i}{\partial \xi^i}, & i = 1, 2 \\
\frac{\partial \tilde{u}^3}{\partial \xi^i} + \Gamma^3_i \tilde{u}^i, & i = 3.
\end{cases}
\end{align*}
\]

The determinant of the Jacobian of the transformation,
\[
\frac{\partial \xi^i}{\partial \tilde{x}^j} = \tilde{G}^{ij/2},
\]
is given by
\[
\begin{vmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
\frac{\partial h}{\partial \xi^1} & \frac{\partial h}{\partial \xi^2} & \frac{\partial h}{\partial \xi^3}
\end{vmatrix} = \tilde{G}^{1/2} = \frac{\partial h}{\partial \xi^3} = \frac{\partial h}{\partial \sigma}.
\]

The tangent and normal basis vectors for the generalized vertical coordinate system in terms of the rectangular representation are given by
\[
\begin{align*}
\mathbf{\tau}_1 &= \mathbf{i} + k \frac{\partial h}{\partial \xi^3}, \quad \eta^1 = i \\
\mathbf{\tau}_2 &= j + k \frac{\partial h}{\partial \xi^2}, \quad \eta^2 = j \\
\mathbf{\tau}_3 &= k \frac{\partial h}{\partial \xi^3}, \quad \eta^3 = 1 \frac{\partial \sigma}{\partial \xi} + j \frac{\partial \sigma}{\partial \eta} + k \frac{\partial \sigma}{\partial \zeta}
\end{align*}
\] (4)

The kinetic energy is computed from these expressions by
\[
e = \frac{1}{2}(\tilde{u}^1 \tilde{u}_1 + \tilde{u}^2 \tilde{u}_2 + \tilde{u}^3 \tilde{u}_3).
\]

As mentioned earlier, averaging of (3) is required if these equations are to be used in meteorological numerical models with finite grid and time intervals. The correct form of averaging this equation is called the grid-volume average and is given by
\[
\int_t \int_{z_1} \int_{z_2} \int_{\sigma} d\sigma d\tilde{x}^3 d\tilde{x}^1 dt((\Delta \tilde{x}^1)(\Delta \tilde{x}^3)(\Delta \sigma)(\Delta t)).
\] (5)

In deriving this form it has been assumed that
\[
\theta = \hat{\theta}(1 + \theta \theta^{-1}) = \hat{\theta} \text{ and that}
\]
\[
\frac{\partial \tilde{u}^i}{\partial t} = \tilde{u}^i;
\]
\[
\frac{\partial \tilde{u}_i}{\partial t} = \frac{\partial \tilde{u}_i}{\partial t}, \text{ etc. (therefore } \tilde{u}^0 = 0, \text{ etc.).}
\] (7)

Assumption (7), when applied in a rectangular coordinate system, is called Reynolds's averaging. To make this assumption in the transformed coordinate system, however, it is necessary to require that changes of the metric tensor over the four-dimensional grid-volume \(\Delta \tilde{x}^1 \Delta \tilde{x}^3 \Delta \sigma \Delta t\) are small, since this tensor appears in (6). Expressed mathematically, this requirement can be written as

This requirement has significant implications on the choice of the vertical generalized coordinate since it must be selected such that variations of the grad-
The advection term in (6) is derived from
\[ \begin{align*}
\overline{\ddot{u} \dot{u}}_t &= \ddot{u} \frac{\partial \ddot{u}}{\partial \dot{x}} + \ddot{u} \frac{\partial \ddot{u}}{\partial \dot{y}} - \dot{u} \frac{\partial \ddot{u}}{\partial x} - \dot{u} \frac{\partial \ddot{u}}{\partial y} + \ddot{u} \frac{\partial \ddot{u}}{\partial \dot{z}} + \ddot{u} \frac{\partial \ddot{u}}{\partial z} \\
&= \ddot{u} \frac{\partial \ddot{u}}{\partial \dot{x}} + \ddot{u} \frac{\partial \ddot{u}}{\partial \dot{y}} + \ddot{u} \frac{\partial \ddot{u}}{\partial \dot{z}} + \ddot{u} \frac{\partial \ddot{u}}{\partial z},
\end{align*} \]
where the assumption that changes of the metric tensor and its derivatives are small permits the removal of the Christoffel symbol from the integrand [this assumption can also be written as]
\[ \dot{\Gamma}_{ij} = \dot{\Gamma}_{ij} + \ddot{\Gamma}_{ij} = \ddot{\Gamma}_{ij} \equiv \ddot{\Gamma}_{ij} \]
where
\[ |\ddot{\Gamma}_{ij}| \ll |\ddot{\Gamma}_{ij}|. \]
The Coriolis term can be expanded as
\[ \begin{align*}
\frac{\partial \ddot{u}}{\partial t} &= \ddot{u} \frac{\partial \ddot{u}}{\partial \dot{x}} - \dot{u} \frac{\partial \ddot{u}}{\partial \dot{x}} - \dot{u} \frac{\partial \ddot{u}}{\partial \dot{y}} + \ddot{u} \frac{\partial \ddot{u}}{\partial \dot{z}} + \ddot{u} \frac{\partial \ddot{u}}{\partial z} + \left( \dot{f} + \frac{\partial h}{\partial \dot{\chi}} \ddot{u} \right) \\
&= \ddot{u} \frac{\partial \ddot{u}}{\partial \dot{x}} - \dot{u} \frac{\partial \ddot{u}}{\partial \dot{x}} - \dot{u} \frac{\partial \ddot{u}}{\partial \dot{y}} + \ddot{u} \frac{\partial \ddot{u}}{\partial \dot{z}} + \ddot{u} \frac{\partial \ddot{u}}{\partial z} + \left( 1 + \left( \frac{\partial h}{\partial \dot{\chi}} \right)^2 \right) \ddot{u}^2 + \frac{\partial h}{\partial \dot{\chi}} \ddot{u}^3, \tag{8}
\end{align*} \]
\[ \begin{align*}
\frac{\partial \ddot{u}}{\partial t} &= \ddot{u} \frac{\partial \ddot{u}}{\partial \dot{x}} - \dot{u} \frac{\partial \ddot{u}}{\partial \dot{x}} - \dot{u} \frac{\partial \ddot{u}}{\partial \dot{y}} + \ddot{u} \frac{\partial \ddot{u}}{\partial \dot{z}} + \ddot{u} \frac{\partial \ddot{u}}{\partial z} + \left( 1 + \left( \frac{\partial h}{\partial \dot{\chi}} \right)^2 \right) \ddot{u}^2 + \frac{\partial h}{\partial \dot{\chi}} \ddot{u}^3, \tag{9}
\end{align*} \]
\[ \begin{align*}
\frac{\partial \ddot{u}}{\partial t} &= \ddot{u} \frac{\partial \ddot{u}}{\partial \dot{x}} - \dot{u} \frac{\partial \ddot{u}}{\partial \dot{x}} - \dot{u} \frac{\partial \ddot{u}}{\partial \dot{y}} + \ddot{u} \frac{\partial \ddot{u}}{\partial \dot{z}} + \ddot{u} \frac{\partial \ddot{u}}{\partial z} + \left( 1 + \left( \frac{\partial h}{\partial \dot{\chi}} \right)^2 \right) \ddot{u}^2 + \frac{\partial h}{\partial \dot{\chi}} \ddot{u}^3 \tag{10},
\end{align*} \]
To illustrate the effect of utilizing the hydrostatic assumption in (8)–(10), it is convenient to use (1) as the generalized vertical coordinate. The relation between the spatial coordinates in the two representations is given by
\[ \begin{align*}
\hat{x}^1 &= x, \\
\hat{x}^2 &= y, \\
\hat{x}^3 &= \sigma = s[z - z_o(x, y)]/(s - z_o(x, y)), \\
z &= h = \frac{\sigma}{s} [s - z_o(\hat{x}^1, \hat{x}^2)] + z_o(\hat{x}^1, \hat{x}^2) \tag{11},
\end{align*} \]
so that the nonzero quantities needed to evaluate the Jacobian, metric tensor and Christoffel symbol are given as

\[
\begin{align*}
\frac{\partial \sigma}{\partial x} &= \frac{\partial z_G}{\partial x} \left( \sigma - s \right), \quad \frac{\partial h}{\partial \tilde{x}} = \frac{\partial z_G}{\partial \tilde{x}^1} \frac{\partial (s - \sigma)}{s} \\
\frac{\partial \sigma}{\partial y} &= \frac{\partial z_G}{\partial y} \left( \sigma - s \right), \quad \frac{\partial h}{\partial \tilde{x}^2} = \frac{\partial z_G}{\partial \tilde{x}^2} \frac{\partial (s - \sigma)}{s} \\
\frac{\partial \sigma}{\partial z} &= \frac{s - \sigma}{s - z_G}, \quad \frac{\partial h}{\partial \sigma} = \frac{\partial z_G}{\partial \sigma} s - z_G,
\end{align*}
\] (12)

as long as the magnitude of \( \partial \tilde{\tau}/\partial \tilde{x}^k \) is at least as large as that of \( \partial \tilde{\tau}/\partial \tilde{x}^1 \) and \( \partial \tilde{\tau}/\partial \tilde{x}^2 \).

If the hydrostatic assumption is applied, where acceleration in the \( \sigma \) direction [which is essentially vertical as given by (14)] and the Coriolis terms are much less than the pressure gradient and the gravitational acceleration terms, then (15) reduces to

\[
\frac{\partial \tilde{\tau}}{\partial \tilde{x}^3} = \frac{\partial \tilde{\tau}}{\partial \tilde{z}} = - \frac{g}{\tilde{\theta}} \frac{\partial \tilde{z}}{\partial \tilde{z}} = - \frac{g}{\tilde{\theta}} \frac{s - z_G}{s}.
\] (16)

Similarly, (8) and (9) reduce to

\[
\frac{\partial \tilde{u}^i}{\partial t} = \tilde{u}^j \frac{\partial \tilde{u}^i}{\partial \tilde{x}^j} - \tilde{u}^p \frac{\partial \tilde{u}^i}{\partial \tilde{x}^p} - \frac{\partial \tilde{\tau}}{\partial \tilde{x}^i} + g \frac{\sigma - s}{s} \frac{\partial z_G}{\partial x} + \tilde{f} \tilde{u}^i - \tilde{f} \tilde{u}^i,
\] (17)

\[
\frac{\partial \tilde{u}^i}{\partial t} = \tilde{u}^j \frac{\partial \tilde{u}^i}{\partial \tilde{x}^j} - \tilde{u}^p \frac{\partial \tilde{u}^i}{\partial \tilde{x}^p} - \frac{\partial \tilde{\tau}}{\partial \tilde{x}^i} + g \frac{\sigma - s}{s} \frac{\partial z_G}{\partial y} + \tilde{f} \tilde{u}^i - \tilde{f} \tilde{u}^i.
\] (18)

Eqs. (16)–(18) are in the form obtained when the chain rule is applied separately for the horizontal and the vertical equations of motion. As shown here, however, (16)–(18) are only approximate relationships when a complete tensor transformation is applied, and they are valid only when (14) applies. The terms given in (14) can also be written as

\[
\left| \frac{\partial \sigma}{\partial x} \right|_{\text{max}} = \left| \frac{\partial z_G}{\partial x} \right|, \quad \left| \frac{\partial \sigma}{\partial y} \right|_{\text{max}} = \left| \frac{\partial z_G}{\partial y} \right|.
\]

Hence, the inequality given by (14) states that \( |\partial z_G/\partial x| = |\partial z_G/\partial y| \ll 1 \) is a necessary condition to assure the validity of (16)–(18). In terms of the terrain representation, this condition requires that the slope must have an angle \( \ll 45^\circ \).

The subgrid-scale terms which are included in (17) and (18) must also be parameterized in terms of known quantities in order to completely specify these equations. In the original rectangular coordinate system, it is customary practice to decompose the subgrid-scale terms into vertical and horizontal components, such that, for the equation of motion with \( i = 1 \), for example,

\[
- \tilde{\rho} \tilde{u}^\tau \tilde{u}^\tau = F_{z} \tilde{u}, \quad - \tilde{\rho} \tilde{u}^\tau \tilde{u}^\tau = F_{y} \tilde{u}, \quad (j = 1, 2),
\]

where \( F_{z} \) represents the vertical turbulent fluxes of the east−west, \( u \), component of velocity, while \( F_{y} \) indicates the horizontal turbulent fluxes of \( u \) (i.e., \( -\tilde{\rho} u^\tau \tilde{v}^\tau \) and \( -\tilde{\rho} \tilde{v}^\tau \tilde{u}^\tau \)). This separation into two components in mesoscale models is necessitated for two major reasons:
In most mesoscale models, the horizontal grid spacing ($\Delta x$, $\Delta y$) is much larger than the vertical spacing ($\Delta z$) so that the parameterizations of subgrid scale mixing in the horizontal and vertical directions would be expected to be quite different.

Much more is known about the functional form of vertical subgrid scale fluxes than of horizontal subgrid-scale fluxes. Thus, two completely different parameterizations are required, with the vertical flux representation being much more detailed.

In a terrain-following coordinate system, when

$$\left| \frac{\partial z_G}{\partial x} \right| \approx \left| \frac{\partial z_G}{\partial y} \right| \ll 1,$$

it, therefore, is desirable to retain this separation into vertical and horizontal flux components. To illustrate this, multiply the first two terms on the right-hand side of the equality in (17) by $\hat{\rho}(s - z_G)/s$ so that

$$\hat{\rho} \left( \frac{s - z_G}{s} \right) \left[ \left( \hat{u} + \hat{v}\right) \frac{\partial}{\partial \hat{z}} \left( \hat{u} + \hat{v}\right) \right],$$

$$= \hat{\rho} \left( \frac{s - z_G}{s} \right) \left[ \left( \hat{u} + \hat{v}\right) \frac{\partial}{\partial \hat{z}} \left( \hat{u} + \hat{v}\right) \right],$$

$$= \frac{\partial}{\partial \hat{z}} \left( \frac{s - z_G}{s} \right) \left[ \left( \hat{u} + \hat{v}\right) \frac{\partial}{\partial \hat{z}} \left( \hat{u} + \hat{v}\right) \right],$$

In writing this expression, the anelastic form of the conservation of mass equation, [i.e., $\partial(\rho u_x)/\partial x = 0$] in the transformed system, given by

$$\frac{1}{s - z_G} \frac{\partial}{\partial \hat{z}} \left( \frac{s - z_G}{s} \right) = 0,$$

along with the assumption that $u = v = 0$ as defined by (7), has been used. Similar terms, of course, can be derived for the advection terms in (18).

Thus, in the transformed coordinate system the subgrid-scale fluxes are given as

$$\hat{\rho} \left( \frac{s - z_G}{s} \right) \hat{u} \hat{v} \hat{u} = F_{\sigma u},$$

$$\hat{\rho} \left( \frac{s - z_G}{s} \right) \hat{u} \hat{v} \hat{v} = F_{\sigma v},$$

where $F_{\sigma u}$ and $F_{\sigma v}$ are, respectively, the $\hat{u}$ fluxes in the $\hat{z}$ direction, and the $\hat{x}$ and $\hat{z}$ directions.

Since it is assumed that

$$\frac{\partial \sigma}{\partial x} \approx \frac{\partial \sigma}{\partial x^2} \ll 1,$$

and so

$$\frac{\partial \sigma}{\partial z} \approx 1,$$

it is reasonable to also assume that the fluxes in the $\hat{x}$ and $\hat{z}$ directions in the two systems are almost equal and so

$$F_{\sigma u} = F_{\sigma v} = \hat{\rho} \hat{u} \hat{v} \hat{u} = \hat{\rho} \left( \frac{s - z_G}{s} \right) \hat{u} \hat{v} \hat{u} = \frac{s}{s - z_G} \hat{u} \hat{v} \hat{u},$$

where the overbar with the $R$ superscript is used to emphasize that this averaging volume is different from that given by (5) (in this case a rectangular volume). Moreover, if $\hat{u} \hat{v} \hat{u}$ is assumed proportional to an exchange coefficient which is a function of height $\xi$ above the ground and the mean velocity profile $\hat{u}$, as is often done, then

$$\hat{u} \hat{v} \hat{u} = \frac{s}{s - z_G} \hat{u} \hat{v} \hat{u},$$

Since $\hat{u} = \hat{u}$, $\xi = \sigma(s - z_G)/s$ and $(\partial/\partial z) = s/(s - z_G) \times \partial/\partial \hat{z}$, then this approximation for the vertical subgrid-scale flux becomes

$$\hat{u} \hat{v} \hat{u} = -\left( \frac{s}{s - z_G} \right) K \left( \frac{s - z_G}{s} \right) \hat{u} \hat{v} \hat{u},$$

and so $\hat{x}$ flux term in (17) can be represented as

$$\hat{u} \hat{v} \hat{u} \hat{v} = \left( \frac{s}{s - z_G} \right)^2 \frac{\partial \hat{u} \hat{v} \hat{u}}{\partial \hat{z}},$$

where $K$ is a function of $(s - z_G)/s$ (i.e., a function of height above the ground). The subgrid flux in the $\hat{x}$ direction in (18) can be shown to have the same form.

The subgrid-scale fluxes in the $\hat{x}$ and $\hat{z}$ directions could be written in a similar form; however, since essentially nothing is known about their functional form on the mesoscale in the rectangular coordinate system, no purpose is served by writing them here. Subgrid-scale fluxes in the horizontal direction are included in models for computational reasons only.

3. Summary and discussion

This paper uses tensor transformation procedures in order to derive a terrain-following coordinate system which is frequently used in a number of regional and mesoscale hydrostatic models. The technique utilizes tensor transformation procedures in order to ensure the physical invariance of the con-

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3 See Dutton (1976, p. 143) for the derivation of this mass conservation relation in any holonomic coordinate system.
servation relations between the Cartesian and
terrain-following systems.

The analysis has shown that, in general, applying the chain rule separately to the hydrostatic equa-
tion and the horizontal equations of motion in order
to transform them to a generalized vertical coordi-
nate system yields a different form of equation than
when the tensor transformation is applied before
the hydrostatic assumption is made. Only when the
slope of the terrain is much less than 45° will the
two procedures of obtaining transformed equations
yield the same forms.

In addition, if the hydrostatic assumption is made
before the coordinate transformation is applied, ki-
netic energy is generally calculated by the contra-
viant velocity components (i.e., \( e = \frac{1}{2} a^i \ddot{a}_i \)).
As
given by Dutton (1976, p. 250), however, this formu-
lation is incorrect since kinetic energy should be
derived from the product of the contravariant and
covariant velocity components (i.e., \( e = \frac{1}{2} (a^i \ddot{a}_i) \)).
Even for small slope angles, the use of the contra-
viant components alone could result in significant
errors in kinetic energy calculations.

Moreover, because the equations must be av-
erged, in order to apply them to mesoscale and
regional-scale atmospheric problems, variations of
the gradient in the generalized vertical coordinate
system within a grid volume must be small. When
the generalized coordinate includes terrain eleva-
tion, this condition requires that the variations
in the gradient of the actual topography within areas
equivalent to the model grid mesh, must be much
smaller than variations in the averaged terrain slope
between adjacent grid mesh areas. Otherwise, terms
such as the subgrid scale velocity fluxes must in-
clude the effects of correlations between terrain
slope and the fluctuating velocities.

The Fourier decomposition of elevation over a
region offers one methodology to examine terrain
variability over a region. Pielke and Kennedy (1980),
have recently used such a spectral technique in
order to determine the spatial scales of the ter-
rain slopes in a portion of central Virginia. Such an
evaluation is essential for the definition of the proper
grid resolution required in a model (i.e., to assure
that most variations of terrain slope are on a spatial
scale larger than the grid increment in the model).

It was also shown that the effect of the general-
ized coordinate on the parameterization of the plan-
etary boundary layer must be considered. In defining
a profile exchange coefficient as a function of distance
above the ground, for instance, the eddy coefficient
must be computed from the actual height and not the
new generalized vertical coordinate. Although
boundary-layer theory over irregular terrain is not
well advanced, and though errors in its proper represen-
tation using horizontally homogeneous, steady-
state boundary-layer theory may be significant, one
should at least be assured that its mathematical
representation is consistent.

Finally, this study was undertaken in order to
critically examine the use of a terrain-following co-
dordinate system in hydrostatic models. Although the
resultant equations usually can be written in the
same form as those used in the past, this investiga-
tion critically delineates the conditions under which
they apply. These conditions should be considered
whenever simulations of atmospheric circulations
are attempted.

Acknowledgment. The authors wish to thank
Klaus-Peter Hoinka for originally motivating our
interest in this study. An exchange of corre-
respondence and several visits to Virginia by Dr.
Hoinka were very beneficial in posing questions
which were investigated during this study. Dr.
Michael McCumber is appreciatively acknowledged
for reviewing the manuscript. Professor John A.
Dutton, Dr. Tzvi Gal-Chen and Dr. Norman Phillips
are thanked for their constructive criticism during
the review of this paper and Ann Gaynor and Susan
Grimstead are thanked for the excellent job of pre-
paring the manuscript and for performing valuable
editorial service.

The work was supported by the Atmospheric Sci-
ences Section of the National Science Foundation.

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