Reply

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I am glad that Drs. Wong and Hage have read the Pielke and Martin (1981) paper in depth; however, they have misinterpreted one of the main conclusions in our 1981 analysis. They have also neglected to refer to Dutton's (1976, Section 7.4) discussion regarding the influence on a consistent tensor transformation of the assumptions of hydrostatic balance and of quasi-horizontal flow. In order to reiterate our result, I will summarize below some of the needed information as well as clarify one statement concerning the orientation of σ-surfaces.

As discussed by Dutton (1976, p. 130), the tangent basis vector is tangent to the curve along which only $\tilde{x}^i$ varies. Therefore, in the transformed coordinate system discussed in Pielke and Martin,

$$\left.\begin{array}{l}\tilde{x}^1 \text{ is in the direction } \mathbf{i} + \mathbf{k}\partial h/\partial \tilde{x}^1 \\ \tilde{x}^2 \text{ is in the direction } \mathbf{j} + \mathbf{k}\partial h/\partial \tilde{x}^2 \\ \tilde{x}^3 \text{ is in the direction } \mathbf{k}\partial h/\partial \tilde{x}^3\end{array}\right\}, \quad (1)^1$$

where $h = (\sigma/s)[s - z_G] + z_G$ as given by Eq. (11) in that paper. Since $\tilde{x}^3 = \sigma$, the σ-coordinate is vertical.

The velocity vector in the transformed system can be represented in terms of either the tangent basis vectors (i.e., $\boldsymbol{\tau}_i$), or the normal basis vectors (see p. 1709 of Pielke and Martin, 1981). In terms of the tangent basis vectors[2]

$$\mathbf{V} = \tilde{u}^i \boldsymbol{\tau}_i,$$

where $\tilde{u}^i$ are the contravariant velocity components. Eqs. (8)–(10) in Pielke and Martin are written in terms of these contravariant velocity components. Therefore, the spatial direction in terms of the Cartesian basis vectors, **i**, **j** and **k** of the contravariant velocity components can be determined from the tangent basis vector [i.e., Eq. (1) in this reply].

As evident from (1), $\tilde{u}^1$ and $\tilde{u}^2$ are at some angle to the horizontal when terrain slope exists, whereas $\tilde{u}^3$ *is in the vertical direction only,* irrespective of terrain slope. Thus, while the magnitudes of $\tilde{u}^1$ and $\tilde{u}^2$ are the same as the Cartesian velocities u and v (see p. 1711 of Pielke and Martin), their directions are not.

One purpose of our analysis was to show that a tensorial transformation of the equations of motion to a terrain-following coordinate system results in three component equations for the contravariant velocity components—with $\tilde{u}^1$ and $\tilde{u}^2$ being terrain parallel at the surface and $\tilde{u}^3$ vertical. Since $\tilde{u}^3$ is vertical, if vertical accelerations in terms of $\tilde{u}^3$ are considered small relative to the other terms in (10) of Pielke and Martin, a hydrostatic equation results in the σ-direction. This is the correct form of the hydrostatic equation if accelerations parallel to σ-surfaces (i.e., $\sim$ terrain parallel accelerations near the ground) are to be included but accelerations in the σ-direction are not. In steep terrain, velocities defined in terms of a terrain-following coordinate rather than a Cartesian representation may be more desirable [e.g., see the discussion by Mahrt (1982, p. 2703)]. In fact, by permitting accelerations parallel to σ surfaces, nonhydrostatic pressure effects which act in that direction are included using this coordinate representation (i.e., $\tilde{u}^1$ has a component in the **k** direction when terrain slope exists). In our paper we demonstrated that these two forms reduce to the same hydrostatic assumption only when the slope becomes flat.

Wong and Hage, on the other hand, correctly conclude that the hydrostatic equation written in terms of the Cartesian velocity components does not depend on terrain slope. In their analysis, u and v remain perpendicular to the gravity vector so that the only vertical direction is contained in the w equation. Therefore, their analysis maps the individual velocity components u, v and w onto an $\tilde{x}^1$–$\tilde{x}^2$–$\tilde{x}^3$, representation, but retains the original directions of the Cartesian velocities. Our analysis, on the other hand, was designed to investigate the effect of transforming these velocities before a hydrostatic assumption was applied.

[1] See Pielke and Martin, 1981, p. 1709.
[2] See Dutton, 1976, p. 130.

And, since none of the velocities are perpendicular to the gravity vector (except for zero slope), we defined the hydrostatic relation to involve the elimination of acceleration in the σ-direction (which is in the k-direction), but we permitted σ-surface acceleration to remain.

Therefore, in conclusion, we agree with Wong and Hage that our analysis involved the assumption that the acceleration of the transformed vertical velocity $\tilde{u}^3$, rather than that of the original Cartesian vertical velocity w, is negligible in the hydrostatic limit. That was one purpose of our analysis—in which a more generalized hydrostatic assumption resulted. Whether the use of transformed velocities such as derived in Pielke and Martin (1981), or a Cartesian representation such as discussed by Hage and Wong (1983) proves to be more useful in the application and interpretation of model results over steep terrain still needs to be investigated.

Finally, Wong and Hage are correct that small slopes do not guarantee the adequacy of the hydrostatic assumption (in either form), while steep slopes do not preclude its validity under strong thermodynamic stability conditions. Over flat terrain in a sea- and land-breeze situation, for example, Martin and Pielke (1983) found the hydrostatic assumption to become more accurate as the synoptic thermodynamic stability increased. The results of that study are likely to generalize to stability effects in complex terrain as well.

I appreciate the opportunity to respond to the comments of Wong and Hage, and hope our exchange has clarified the intent of the Pielke and Martin (1981) paper. Pielke (1984) promotes an additional discussion of the hydrostatic assumption when applied to this terrain-following coordinate system.

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