A SUGGESTED REFINEMENT FOR O'BRIEN'S CONVECTIVE BOUNDARY LAYER EDDY EXCHANGE COEFFICIENT FORMULATION

(Research Note)

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Abstract. With observational data collected and interpreted by Crane *et al.* (1977), the adequacy of the O'Brien polynomial to represent the exchange profile of heat and pollution in a convective boundary layer is examined and a refinement suggested. Also, it is shown that the height of the surface layer, $h = 0.04\,z_i$, developed by Blackadar and Tennekes (1968) for a neutrally stratified boundary layer (with $z_i = 0.25\,u_*/f$) appears to be equally valid for the convective boundary layer where z_i, defined as the top of the mixed layer, is used.

1. Introduction

O'Brien (1970) introduced an interpolation formula to represent mixing within the planetary boundary layer, above the surface layer. His basic intent was to apply a cubic polynomial to the vertical turbulent eddy exchange coefficient which had a specified magnitude and vertical gradient at the top of the surface layer (K_h and $\partial K/\partial z|_h$, both of which are positive) and a zero gradient and specified magnitude, K_{z_i}, at the top of the planetary boundary layer. The form of this profile function, given by

$$K(z) = K_{z_i} + [(z_i - z)^2/(z_i - h)^2]\,\{K_h - K_{z_i} + (z - h)[\partial K/\partial z|_h + {} + 2(K_h - K_{z_i})/(z_i - h)]\}\,, \tag{1}$$

has been used successfuly by a number of investigators including Pielke and Mahrer (1975), Tapp and White (1976), Yu (1977) and others.

As applied by Pielke and Mahrer (1975), for an unstable or neutrally stratified surface layer, the top of the mixed layer z_i is determined by the prognostic equation introduced by Deardorff (1974). The value of K at the top of the surface layer, h, and the vertical gradient of K at that level, $\partial K/\partial z|_h$, are determined using the semi-empirical similarity formulas for the nondimentisonal temperature gradient, ϕ_H, discussed by Businger *et al.* (1971) and Businger (1973). As implied by the results of Yamada (1977), ϕ_H could represent the appropriate nondimensional pollution vertical gradient as well.

For a neutrally and unstably stratified surface layer (i.e., $z/L \leq 0$), the form of ϕ_H is given by

$$\phi_H = 0.74[1 - 9z/L]^{-1/2} \tag{2}$$

where $L = \bar{\theta} u_*^2/\mathrm{kg}\theta_*$.

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The purpose of this paper is to examine the consistency of the O'Brien polynomial parameterization for the diffusion of pollution for convective boundary layers where $|z/L| \gg 1$. The data reported by Crane *et al.* (1977) are used for this comparison.

2. Comparison Against Data

Within the surface layer:

$$K = ku_* z/\phi_H\,, \tag{3}$$

where u_* is the friction velocity and k is von Karman's constant. For the situation of large surface layer instability (i.e., large $|z/L|$), from (2), Equation (3) can be written as†

$$K = -4.1\,k^{3/2} z^{3/2} (g\theta_*/\bar{\theta})^{1/2}\,. \tag{4}$$

Assuming $K_{z_i} = 0$, (1) can be rewritten as

$$\frac{K}{K_h} = \frac{(1 - z/z_i)^2}{(1 - h/z_i)^2}\left\{1 - (z - h)\left[\frac{1}{h} - \left.\frac{\partial \ln \phi_H}{\partial z}\right|_h + \frac{2}{z_i - h}\right]\right\} \tag{5}$$

where $(\partial K/\partial z)|_h = (K_h/h) - K_h(\partial \ln \phi_H/\partial z)$, obtained from (3) has been used. We are writing K in this format in order to compare against the observational deduction of K as reported by Crane *et al.* (1977).

Assuming $z_i \gg h$ (Blackadar and Tennekes (1968) found that for the neutral boundary layer, $h = 0.04\, z_i$ yielded the best results), (5) can be simplified to

$$\frac{K}{K_h} = \frac{(1 - \alpha)^2}{(1 - \beta)^2}\left[1 + (\alpha - \beta)\left(\frac{3}{\beta} + \frac{2}{1 - \beta}\right)\right], \tag{6}$$

where $\alpha = z/z_i$ and $h = \beta z_i$ has been used. Also, since $|z/L| \gg 1$ is assumed, the second term within the brackets of (5) is given by

$$\left.\frac{\partial \ln \phi}{\partial z}\right|_h = -\left.\frac{1}{2z}\right|_h = -\frac{1}{2h}\,.$$

Below $z = h$, from (4)

$$\frac{K}{K_h} = \left(\frac{z}{h}\right)^{3/2}. \tag{7}$$

† As explained by Carl *et al.* (1973), K should be proportional to $z^{4/3}$ in the limit of very large negative z/L. This occurs because K should become independent of wind speed when the surface layer is sufficiently unstable. In order to apply (1) to a model, however, we have chosen to retain K as given by (4) even though u_* remains in the parameterization through the θ_* term. We recognize the inconsistency at the asymptotic limit but conclude that $K \sim z^{3/2}$ and $K \sim z^{4/3}$ differences cannot be distinguished in the data (i.e., Figure 1a) and that there will not be any significant differences in the parameterization of mixing throughout the boundary layer in a mesoscale model when (1) is used.

The observations, and deduced estimates of K, expressed in terms of K/K_h with $\beta = 0.04$ from Crane *et al.* (1977) are given in Figure 1a, while K/K_h as given by (7) and (6) with $\beta = 0.04$ is illustrated in Figure 1b. As reported by O'Brien and illustrated in Figure 1b, the maximum in (6) is achieved at $\alpha = 0.35$, which appears to be consistent with the observational deduction of Crane *et al.* (1971) given in Figure 1a. In addition, the large values of K in the middle of the convective boundary layer obtained from (6) are consistent with the conclusion from the observations of a very large K in that portion of the boundary layer since such large values of K will result in well-mixed (i.e., ~ zero vertical gradient) profiles.

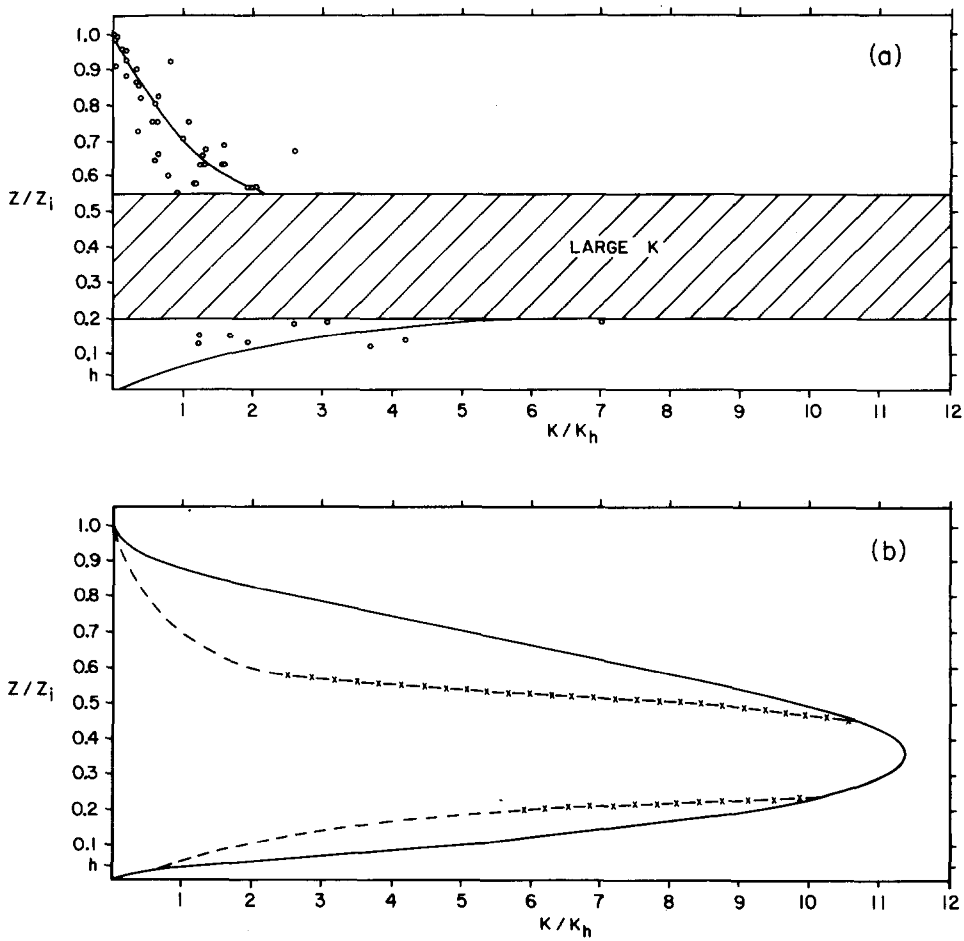

Fig. 1a. Distribution of K/K_h as a function of $\alpha = z/z_i$ as adapted from Crane *et al.* (1977), using $h = 0.04z_i$. (b) Distribution of K/K_h as a function of z/z_i: (1) (solid line) using Equation (6), which is valid for a convective boundary layer (i.e., $|z/L| \gg 1; L < 0$); (2) (dashed line) distribution of K/K_h as a function of z/z_i estimated from the data of Crane *et al.* (1977) and used to create the corrections given by (8). The portion of the dashed line marked by ×'s corresponds to that portion of the profile which was extrapolated from the Crane *et al.* (1977) data in the region of large K as indicated in their figure.

There are some discrepancies, however, between the observed and calculated profiles of K/K_h. Below about $\alpha = 0.2$, the O'Brien profile yields values of K/K_h which are somewhat larger than the observationally deduced values, while the O'Brien K is substantially larger than observed above $\alpha = 0.5$ to 0.9 or so. We could establish a new polynomial expression for K/K_h using the observed data. However, since K/K_h values are not given by Crane *et al.* (1977) for the entire planetary boundary layer, we prefer to correct to a new profile using the O'Brien profile as the basic function. The general pattern of K, however, appears to be well represented by the O'Brien profile which leads us to conclude that the turbulent mixing of pollution and heat within a convective boundary layer can be presented in terms of a K profile, which is defined in terms of height within the mixed layer and the values of K and its derivatives at z_i and h.

We suggest refinements to the O'Brien profile using a quadratic correction in order to achieve the best fit for the data of Crane *et al.* (1977). This correction was achieved by extrapolating the Crane *et al.* (1977) profile, to the same maximum value at the same height as given by the O'Brien formulation. This modified profile is then given by

$$\begin{aligned}
\left(\frac{K}{K_h}\right)_{\mathrm{OBM}} &= \left(\frac{K}{K_h}\right)_{\mathrm{OB}} [28.6\alpha^2 - 8.57\alpha + 1] && 0.04 \le \alpha < 0.3 \\
\left(\frac{K}{K_h}\right)_{\mathrm{OBM}} &= \left(\frac{K}{K_h}\right)_{\mathrm{OB}} && 0.3 \le \alpha \le 0.4 \\
\left(\frac{K}{K_h}\right)_{\mathrm{OBM}} &= \left(\frac{K}{K_h}\right)_{\mathrm{OB}} [9.4\alpha^2 - 13.2\alpha + 4.8] && 0.4 < \alpha \le 1
\end{aligned} \tag{8}$$

where $(K/K_h)_{\mathrm{OB}}$ is the value determined from (1) and $(K/K_h)_{\mathrm{OBM}}$ is the modified value. The modified O'Brien profile is illustrated in Figure 1b, where the $\times$ marks indicate those sections which were extrapolated from the Crane *et al.* (1977) profile.

The use of $h = 0.04z_i$ in (6) and (7) is consistent with the observations of Crane *et al.* (1977). This conclusion differs from that of Aloyan *et al.* (1981) who found that h in unstably stratified air is different from that in a neutral surface layer. Therefore, our relation for h, although introduced by Blackadar and Tennekes (1968) for a neutrally stratified boundary layer, appears to be the appropriate height to use even in a convectively unstable mixed layer.

3. Conclusion

From data collected over the Los Angeles Basin by Crane *et al.* (1977), it is shown that the O'Brien profile representation of the exchange coefficient for heat and pollution is consistent with the observations, although an empirical correction is required in order to obtain the best agreement. In addition, the specification of the top of the surface layer for use in the O'Brien polynomial of $0.04z_i$ provides reasonable results, at least for the data set collected by Crane *et al.* (1977).

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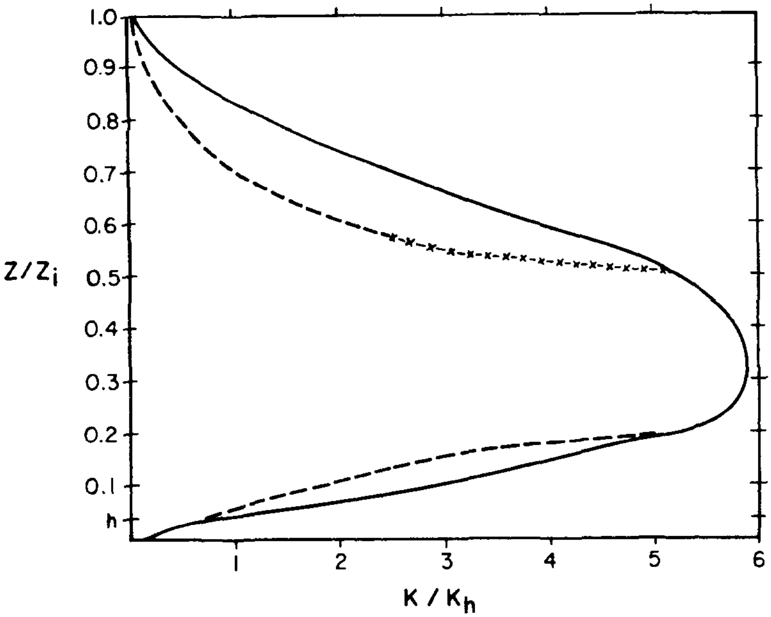
CORRIGENDUM

Pielke, R. A., Panofsky, H. A., Segal, M.: 1983, 'A Suggested Refinement for O'Brien's Convective-layer Eddy Exchange Coefficient Formulation', *Boundary-Layer Meteorology* **26**, 191–195.



Two typographical errors appear in the Note: (i) in Equation (4) the term in the parenthesis should be with a minus sign; (ii) in Equation (5) the minus sign before the $(z - h)$ term should be replaced with a plus sign.

An error appears in Equation (6) where the term $3/\beta$ should be replaced by $3/2\beta$. This error required the redrafting of Figure 1b – a corrected version is given in the Figure of this Corrigendum. As a result of the corrected analysis, Equation (8) has to be readjusted to the following form:

$$\left(\frac{K}{K_h}\right)_{\text{OBM}} = \left(\frac{K}{K_h}\right)_{\text{OB}} [59.42\alpha^2 - 14.26\alpha + 1.47] \qquad 0.04 \le \alpha < 0.2$$

$$\left(\frac{K}{K_h}\right)_{\text{OBM}} = \left(\frac{K}{K_h}\right)_{\text{OB}} \qquad 0.2 \le \alpha \le 0.5$$

$$\left(\frac{K}{K_h}\right)_{\text{OBM}} = \left(\frac{K}{K_h}\right)_{\text{OB}} [10.17\alpha^2 - 15.24\alpha + 6.08] \qquad 0.5 < \alpha \le 1$$

As evident from the Figure, the conclusions regarding the adequacy of the O'Brien polynomial to parameterize the turbulent mixing of heat in a convective boundary layer are further strengthened as a result of this correction.

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