Application of Terrain Height Variance Spectra to Mesoscale Modeling

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ABSTRACT

The distribution of terrain height variance with wavelength is important for determining the required horizontal grid spacing for mesoscale models so that a subgrid-scale terrain parameterization need not be calculated. One-dimensional terrain height variance spectra for western Colorado are calculated from data collected from topographic maps and NOAA/EDIS/NGSDC 30 s average elevation data tapes. The terrain height variance of this region is found to be linearly dependent on wavelength. The Colorado terrain height variance spectra are compared with those calculated for Wales by Bretherton and for Virginia by Pielke and Kennedy. The implications for selection of horizontal grid spacing of mesoscale models in these regions are discussed.

A general conclusion is that successful mesoscale model simulations of actual geographic areas require, as a necessary (although not a sufficient) condition, that terrain variations as a function of wavelength be evaluated for the domain of interest. Such an analysis will indicate the needed grid resolution in order to resolve adequately the entire range of terrain forcing or, alternatively, provide guidance as to how subgrid-scale terrain variations can be parameterized.

1. Introduction

A one-dimensional terrain height variance spectrum is the distribution of terrain height variance with wavelength λ along a cross section of the earth's surface. Terrain height variance spectra are of interest to the mesoscale modeler because they provide information on the dominant wavelengths of terrain variation. This information is used to specify the minimum horizontal grid spacing required to resolve topography without resorting to the parameterization of subgrid-scale terrain variance in numerical models of orographic flows. On the other hand, if parameterization is required because of computer resource limitations, then the functional form of terrain spectra on scales smaller than the model resolution can be used to develop effective parameterizations of such effects as subgrid-scale wave and frictional drag.

Results of previous studies of terrain height spectra vary considerably depending on the geomorphology of the regions studied. Bretherton (1969) calculated terrain height variance spectra for northern Wales in the course of a study of momentum transport by gravity waves. He found terrain height variance to be proportional to $\lambda^{3/2}$ for $\lambda < 30$ km. Pielke and Kennedy (1980) calculated terrain height variance spectra for west-central Virginia. They found terrain height variance to increase as rapidly as λ^2 for short wavelengths. Terrain height variance spectra for three mountainous regions in Colorado were calculated for the current study. The wavelength bands were 0.076 to 337 km, 0.7 to 85 km, and 0.7 to 428 km for the regions referred to as Steamboat Springs, Boulder and South Park, respectively. The observed λ^{+1} dependence of these spectra will be shown to have important consequences in the selection of horizontal grid spacing for mesoscale models of this region.

The terrain height data used in this study were obtained manually from topographical maps of this region and from the NOAA/EDIS/NGSDC 30 s average elevation data tapes described by Dietrich and Childs (1982). The terrain height variance spectra were computed with a computer implementation of the Fast Fourier Transform algorithm. Estimates were made of the percentage of terrain height variance which would be resolved using various horizontal grid spacings in a mesoscale model.

2. Data

Topographic contour maps provide a readily available source of terrain height data on a variety of horizontal scales. Topographic maps on three scales of the region around Steamboat Springs, Colorado were used for this study. Terrain height values were truncated to the value of that contour line which bracketed from below the true height of the data point. This method is considered to be more objective and more economical than interpolating between contour lines. The height errors introduced by this procedure have magnitudes which are randomly distributed between zero and the contour interval of the map used. A second source of data was the NOAA/EDIS/

NGSDC 30 s average elevation tapes. While resolution is limited to 30 s of latitude and longitude, large amounts of data can be acquired much more rapidly from this source than from topographic maps. This data source was used in all three regions.

This study is limited to one-dimensional spectra along the west-to-east direction to demonstrate the procedure. The use of two-dimensional terrain height variance spectra would be preferable in planning three-dimensional numerical modeling experiments but was prohibitively expensive for the large regions to be studied here.

Three data sets were obtained for single cross sections of the earth's surface along the parallel at 40°30'N near Steamboat Springs. The largest scale data set includes 240 height values for points spaced at 1.4 km intervals. These data were obtained from a Sectional Aviation Chart which has a 152 m contour interval. The second largest scale data set is of only a slightly smaller scale than the first. This data set includes 360 height values for points spaced at 0.46 km intervals. These data were obtained from a 1:250 000 scale topographic map which has a 61 m contour interval. The smallest scale data set includes 554 height values for points spaced at 0.038 km intervals. These data were obtained from two 7.5' quadrangle maps which have a 12 m contour interval.

In addition, a data set composed of 20 east–west cross sections spaced north–south at 0.9 km and centered along 40°30'N near Steamboat Springs was acquired from the NOAA/EDIS/NGSDC. Each of these 20 cross sections includes 360 height values for points spaced at 0.7 km intervals. Average spectra derived from this data set are compared with those derived from topographic maps to determine the compatibility of the two data sources.

A data set composed of 20 east–west cross sections spaced north–south at 0.9 km and centered along 40°00'N west of Boulder was also acquired from the NOAA/EDIS/NGSDC tapes. Each of these 20 cross sections includes 120 height values for points spaced at 0.7 km intervals. A similar data set composed of 20 east–west cross sections spaced north–south at 0.9 km and centered along 39°30'N near South Park, Colorado was acquired from the NOAA/EDIS/NGSDC tapes. Each of these 20 cross sections includes 600 height values for points spaced at 0.7 km intervals. Average spectra derived for each of these two data sets are compared with those derived for the Steamboat Springs region to determine the variability of terrain spectra of western Colorado.

3. Method

One-dimensional terrain height variance spectra were computed separately for each of the terrain height cross sections. For each cross section, a linear trend was first calculated by the method of least squares and then subtracted from the terrain height series. The resulting series were transformed into wavenumber space by a computer implementation of the Fast Fourier Transform algorithm. For the data sets containing multiple cross sections, the computed spectra were averaged within each data set. The resulting spectra depict the distribution of terrain height variance with wavenumber or wavelength. The results are stated in metric units. The terrain height variance spectra were plotted against wavenumber and wavelength on a log-log scale to facilitate the derivation of a power law relationship between terrain height variance and wavelength. The method of least squares was then used to fit a relation of the form $S = a\lambda^b$ to each of the three spectra. Confidence intervals for the true values of the exponent B are given. Finally, the value of the exponent of the power law which fits the terrain height variance spectra is used to discuss the choice of horizontal grid spacing for mesoscale models.

TABLE 1. Parameters of the least-squares best fit relation of the form $S = a\lambda^b$ for six terrain height variance spectra for western Colorado. The tabulated parameters are the coefficient a, the exponent b, the correlation coefficient r and the standard deviation s_b of the exponent. The latter three parameters are nondimensional.

Data set	a	b	r	s_b
Steamboat Springs Single cross section, largest scale	0.62	0.93	−0.80	0.06
Steamboat Springs Single cross section, second largest scale	0.45	0.99	−0.82	0.05
Steamboat Springs Single cross section, smallest scale	2.1	0.90	−0.90	0.03
Steamboat Springs Multiple cross section	1.1	0.99	−0.99	0.04
Boulder Multiple cross section	1.8	1.0	−0.99	0.11
South Park Multiple cross section	0.98	0.96	−0.98	0.09

4. Results

As stated in Section 3, relations of the form $S = a\lambda^b$ were fit to the terrain height variance spectra for the six data sets by the method of least squares regression; S is the terrain height variance with units of m^2 km, and λ is the wavelength with units of km.[1] Table 1 displays the values of the coefficient a, the

[1] In order to achieve correct dimensional units, of course, as b changes in this regression formula, the units of a are appropriately adjusted.

exponent b, the correlation coefficient r, and the standard deviation of the exponent s_b for each of the six data sets. The spectra themselves are presented in Figs. 1–6.

Confidence intervals may be computed for the true value of the exponent B using

$$F = \frac{(B - b)^2}{s_b^2},$$

where F has degrees of freedom 1 and $n - 2$, n being the number of harmonics in the spectra (Edwards, 1979). This formula allows $\pm(B - b)$ to be computed from the spectrum. A value of $\alpha = 0.05$ was selected for the computation of confidence intervals. The resulting confidence intervals on the true value of the exponent were $B = 0.93 \pm 0.13$ for the largest scale, single cross section, Steamboat Springs data set; $B = 0.99 \pm 0.10$ for the second largest scale, single cross section, Steamboat Springs data set; and $B = 0.90 \pm 0.05$ for the smallest scale, single cross section, Steamboat Springs data set.

For the multiple cross section data sets, $B = 0.99 \pm 0.09$ for Steamboat Springs, $B = 1.0 \pm 0.29$ for Boulder, and $B = 0.96 \pm 0.18$ for South Park. The values of the exponent for the six data sets are therefore not significantly different from each other at the $\alpha = 0.05$ level.

The coefficient a varies by a factor of 4 between data sets because of differing geographic coverage. This suggests that, although the functional form of the spectral distribution of terrain height variance is similar across western Colorado, the total terrain height variance is not.

Reported terrain height variance spectra for other regions of the world have wavelength dependence with power law exponents ranging from 1.5 to 2.0

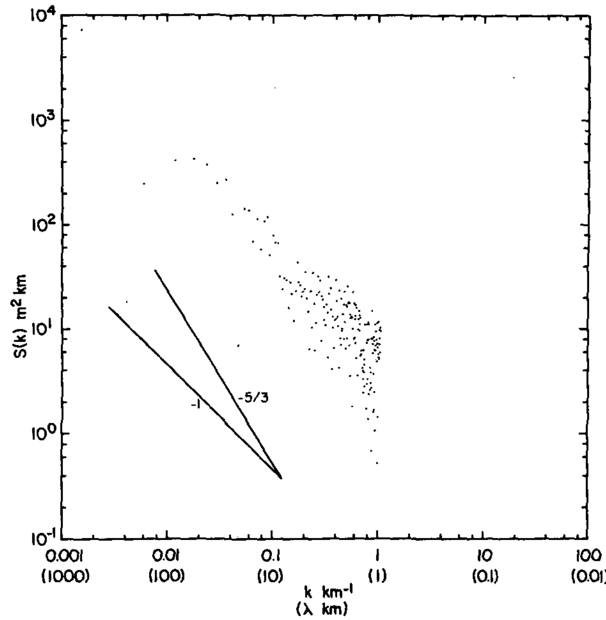

FIG. 2. Terrain height variance spectra for the second largest, single-cross section, Steamboat Springs data set. Data were taken from a 167 km long vertical section along 40°30′N at Steamboat Springs in northwestern Colorado.

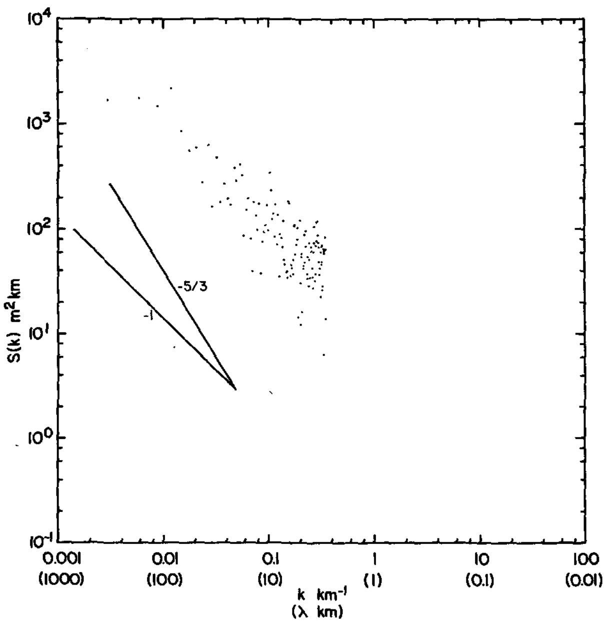

FIG. 1. Terrain height variance spectra for the largest-scale, single-cross section, Steamboat Springs data set. Data were taken from a 333 km long vertical section along 40°30′N at Steamboat Springs in northwestern Colorado.

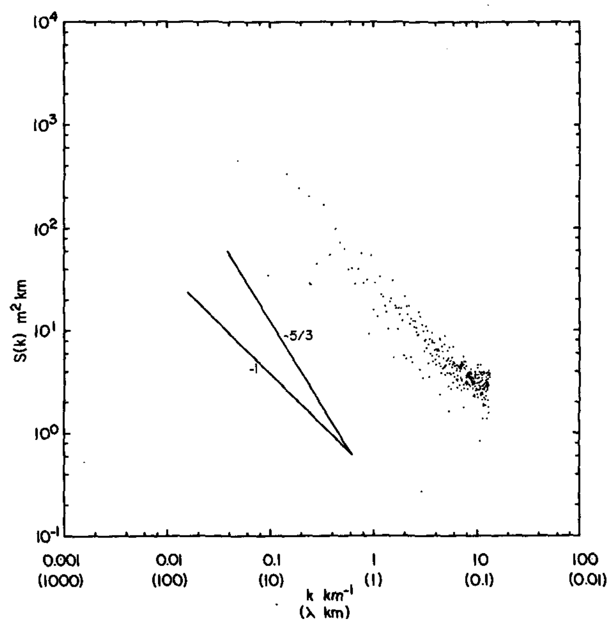

FIG. 3. Terrain height variance spectra for the smallest scale, single-cross section, Steamboat Springs data set. Data were taken from a 34 km long vertical section along 40°30′N at Steamboat Springs in northwestern Colorado.

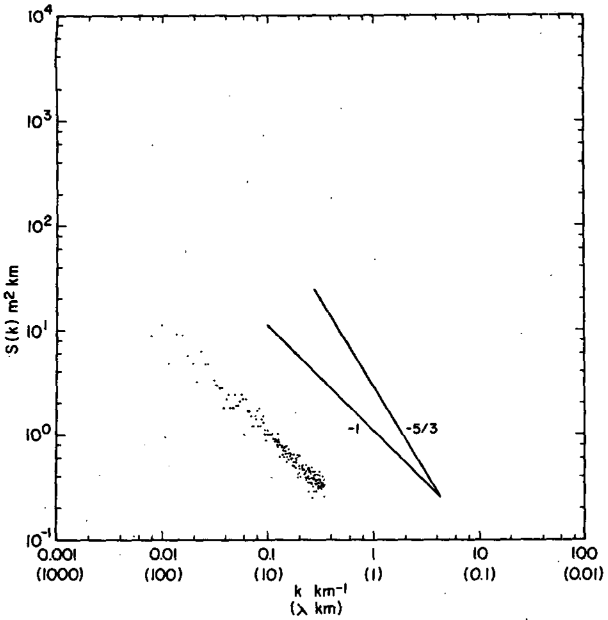

FIG. 4. Average terrain height variance spectra for the multiple-cross section, Steamboat Springs data set. Data were taken from twenty 253 km long, vertical sections centered along 40°30′N at Steamboat Springs in northwestern Colorado.

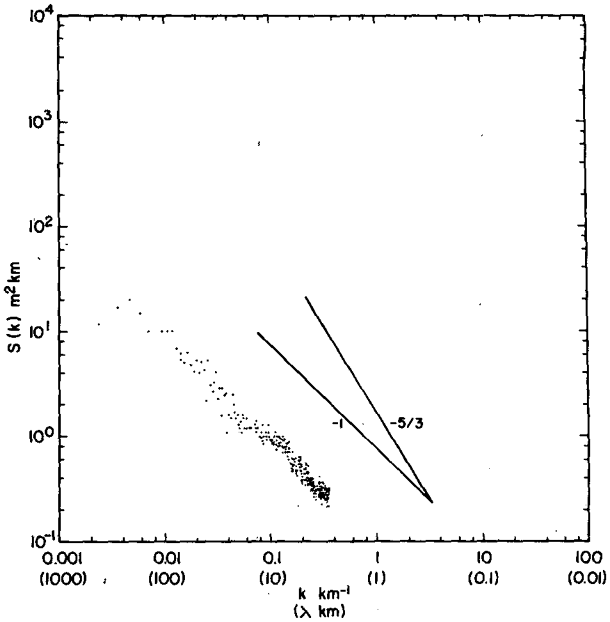

FIG. 6. Average terrain height variance spectra for the multiple-cross section, South Park data set. Data were taken from twenty 428 km long, vertical sections centered along 39°30′N at South Park in western Colorado.

and varying coefficients. Terrain height spectra are therefore dissimilar throughout the world. These include Bretherton (1969) who reported a relation of the form $S = a\lambda^b$ for the average of 90 east–west one-dimensional terrain height variance spectra for north Wales. He found an exponent of 1.5 to fit the spectra for wavelengths < 30 km. Pielke and Kennedy (1980) found terrain height variance spectra of west-central Virginia to have a $\lambda^{5/3}$ dependence at wavelengths near 10 km. The dependence changed smoothly to $\sim\lambda^{9/4}$ for wavelengths $\leqslant$ 4 km.

The exponent of the power law fit to a terrain height variance spectrum provides a quantitative measure of the appearance of terrain "smoothness." Geographic variations in this exponent indicate that relatively smooth appearing older mountain ranges such as those in west-central Virginia have a comparatively smaller proportion of their terrain height variance at short wavelengths than do jagged appearing younger mountain ranges such as those in western Colorado. The use of these terrain height variance spectra will be discussed in the next section.

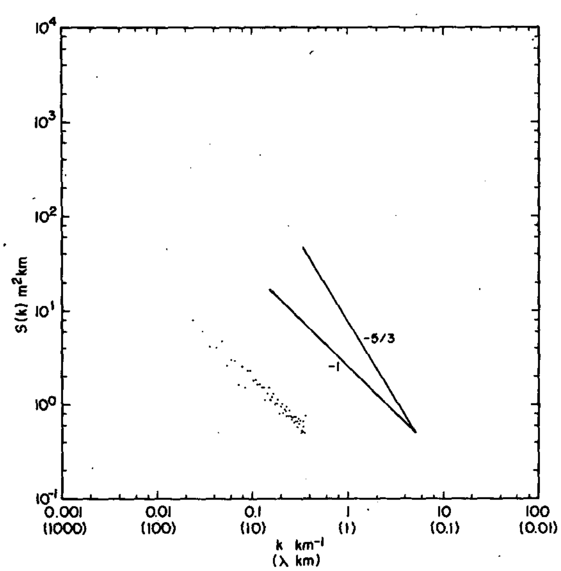

FIG. 5. Average terrain height variance spectra for the multiple-cross section, Boulder data set. Data were taken from twenty 85 km long, vertical sections centered along 40°00′N at Boulder in northwestern Colorado.

5. Discussion

For the purposes of mesoscale modeling, terrain height variations can be divided into two ranges: those with wavelengths greater than $2\Delta x$, which can be resolved by the model,[2] and those with wavelengths less than $2\Delta x$, the subgrid-scale terrain variations. It is desirable to choose a value of Δx, the horizontal grid spacing of the model, small enough so that the effects of subgrid-scale terrain variance are negligible.

Except in linear problems, small-scale terrain can

[2] It should be recognized, of course, that while $2\Delta x$ wavelength features can be resolved, the accuracy of representation of short wavelengths in terms of a numerical model grid is poor (e.g., see Pielke, 1981).

affect larger scale meteorological phenomena.[3] Pielke (1981) states that, for nonlinear mesoscale models using terrain-following coordinates, the terms containing subgrid-scale terrain fluctuations can be neglected in the grid-volume-averaged conservation equations if the subgrid-scale terrain variance is small compared with the grid-resolvable terrain variance. If the terrain variance on scales $< 2\Delta x$ is significant, however, the effect of subgrid-scale terrain variations must be considered in the parameterization of the subgrid-scale flux terms. The presence of subgrid-scale terrain variance should result in enhanced frictional and wave drag from what would be found if this small-scale variance were zero.

Pielke and Kennedy (1980) tabulated minimum wavelengths required to resolve various percentages of the terrain height variance for west-central Virginia. These minimum wavelengths were determined by integrating the area under terrain height spectra and requiring a specified percentage of the terrain height variance to be at wavelengths greater than that minimum. For different cross sections, values of this minimum wavelength ranged from 2.75 to 7.5 km for resolution of 90% of the observed terrain height variations. A similar procedure can be used with any terrain spectra. The resolved terrain height variance is in the wavelength band from $2\Delta x$ to the model domain length while the subgrid-scale terrain height variance is at wavelengths $< 2\Delta x$. The accurate evaluation of subgrid-scale terrain height variance can be accomplished only if the terrain height variance spectra extend to wavelengths short enough so that the contribution of even smaller spatial scales to the terrain height variance is negligible. For any terrain height variance spectra with a power law exponent $\leqslant 1$ at the shortest wavelengths measured, however, the integration to compute the subgrid-scale variance is divergent if one must assume this relation continues at even shorter wavelengths.[4] Thus, to achieve an accurate approximation of subgrid-scale variance, the spectra must extend down to wavelengths at which variance decreases more rapidly than linearly with decreasing wavelength.

The west-central Virginia height variance decreases as $\sim\lambda^{9/4}$ at the shortest wavelengths measured so the subgrid-scale terrain height variance can be approximated well from the finite wavelength band measured. The western Colorado terrain height variance decreases approximately linearly with decreasing wavelength so the subgrid-scale terrain height variance cannot be approximated with the finite wavelength band measured.

One can, however, set an upper bound for the grid spacing required to permit neglect of the subgrid-scale terrain variances. For terrain height variance spectra of the form $S = a\lambda^1$, the ratio of subgrid-scale terrain height variance to model resolved terrain height variance is

$$\frac{\int_{1/2\delta x}^{1/2\Delta x} ak^{-1}dk}{\int_{1/2\Delta x}^{1/n\Delta x} ak^{-1}dk},$$

where $k = 1/\lambda$, $2\delta x$ is the shortest wavelength in the measured spectra, $2\Delta x$ is the shortest wavelength that can be resolved by a numerical model and $n\Delta x$ is the model domain length. This quantity integrates to

$$\frac{\ln(\delta x/\Delta x)}{\ln(2\Delta x/n\Delta x)}.$$

For a mesoscale numerical model of western Colorado with a 100 km domain and using the spectra reported above, $\delta x = 0.076$ km and $n\Delta x = 100$ km. This results in a ratio of subgrid-scale terrain height variance to model-resolved terrain height variance of 0.16 for a grid spacing of 0.1 km. Therefore, 0.1 km is a likely maximum allowable value of horizontal grid spacing for mesoscale models of western Colorado without requiring a parameterization of subgrid-scale terrain effects. If the slow decrease of terrain height variance with decreasing wavelength continues to wavelengths below those measured, the required grid spacing will be smaller.

Thus, numerical models of the mesoscale orographic flows of western Colorado require a much shorter grid spacing to resolve adequately the terrain than do similar models of such flows in west-central Virginia.

Also, although not discussed in detail here, a mesoscale model must either have a domain large enough to include all terrain features which might influence the mesoscale flow, or else include the effects of terrain features outside the domain through appropriate lateral boundary conditions.

The wavelength of maximum terrain height variance can be determined from terrain height spectra. The domain of a mesoscale numerical model must be large enough to encompass features of this dominant wavelength.

6. Conclusions

The terrain height variance spectra calculated for western Colorado have proportionately more power

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[3] Even in linear problems, the contribution of the pressure gradient force to atmospheric flows of a given wavelength are magnified for shorter wavelengths (see Walmsley *et al.*, 1982).

[4] Subgrid-scale terrain height variance is calculated by integrating the terrain height variance spectra from the largest wavenumber k resolvable by the model to ∞ where $k = 1/\lambda$. Integrating the power law used above results in the following form for subgrid-scale terrain height variance:

$$\int_{1/(2\Delta x)}^{\infty} S(k)dk = a\int_{1/(2\Delta x)}^{\infty}\left(\frac{1}{k}\right)^b dk \left.\vphantom{\int_0^\infty}\right\} \begin{array}{l}\text{which is divergent for } b \leqslant 1\\ \text{but convergent for } b > 1,\end{array}$$

where Δx is the grid interval of the numerical model.

at short wavelengths than do those reported for northern Wales and west-central Virginia. The nearly linear dependence of terrain height variance at all measured wavelengths in western Colorado prohibits determination of the exact horizontal grid spacing required to resolve the majority of the terrain height variations. An upper bound of $\Delta x = 0.1$ km for mesoscale numerical models whose conservation equations do not include parameterizations of the terms including subgrid-scale terrain fluctuations was derived for western Colorado. This value is at least an order of magnitude smaller than the required grid spacings found by Pielke and Kennedy (1980) for west-central Virginia.

When computer resources are insufficient to permit the required spatial resolution to be small enough to resolve the significant terrain variations, the influence of the subgrid-scale topographic variance on the grid-volume-averaged subgrid-scale fluxes must be parameterized. The distribution of terrain variance as a function of wavelength on the subgrid scale will aid in this parameterization by indicating how the terrain varies as a function of the subgrid-scale terrain wavelengths. The influence of subgrid-scale variations in terrain on the grid-volume-averaged variables includes enhanced wave and frictional drag.

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