The Extrapolation of Vertical Profiles of Wind Speed within the Marine Atmospheric Surface Layer Using the $p$ Formula

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ABSTRACT

Values of $p$ for the exponent-type wind profile formulation, used in vertical extrapolations of wind speed, were derived for the marine atmospheric surface layer. Nomograms were constructed, providing $p$ values as dependent on a single elevation measurement of the air temperature, wind speed, and the surface water temperature. The range of $p$ values in the unstable surface layer is between 0.02 to 0.2, while for stable situations the range is 0.1 to possibly $\sim 1.0$. The values of $p$ converge to about 0.2 for high wind speeds.

1. Introduction

Extrapolation of wind speed profiles, within the first tens of meters above the land, based on single height wind data, are common in both research and applied studies. The necessity for such extrapolations emerges because of the substantial expenses associated with measured vertical profiles of the wind. Probably the most common extrapolation is based on an exponent type wind profile which is referred to as the wind power formula (e.g., Panofsky and Parsad, 1965; Panofsky and Dutton, 1983)

$$p = \frac{d(\ln u)}{d(\ln z)} = \left( \frac{z}{u} \right)^{\frac{1}{2}} \frac{d^2}{dz^2} \left( \frac{u}{Ri} \right)^{1/2}$$

or on its common approximation through

$$u = u_1 \left( \frac{z}{z_1} \right)^p.$$  \hspace{1cm} (2)

In (2), $u$ refers to the wind speed, $z$ to elevation, the subindex, 1, indicates the reference measurement level, $Ri$ is the Richardson number, $Ri_B$ is the bulk Richardson number as defined by (8), and $p$ is a constant which is a function of the thermal stability in the surface layer. Many studies have been involved with the evaluation of the magnitude of $p$ over a land surface. In recent years, Touma (1977), for example, carried out a comprehensive analysis based on wind data from many sites, to determine by observational means the values of $p$ as a function of the near-surface thermal stability.

The necessity for such extrapolation is even more frequent over large water bodies where vertical profiles of the wind speed are obviously very expensive to acquire. Over the open sea, where even single elevation wind data are rare, utilizing the wind power formula may provide important additional information as to the wind profile within the surface layer. Alternatively, it can provide a formulation for conversion of wind speed measured at different locations and elevations to a standard level. Such evaluations can be obtained, of course, by a detailed computation of the surface layer characteristics (e.g., Smith, 1980; Liu, 1984). However, in analogy to the land case, using the wind power formula approximation over water can be beneficial in many situations (e.g., when a single elevation wind speed is given and the thermal stratification has to be determined subjectively, or when a bulk evaluation is needed).

Unlike the numerous studies relating to $p$ over land surfaces, however, no special attention has been given to a refined investigation of its characteristics over open water surfaces. Davenport (1965) suggested $p \approx 0.1$ as being representative over the open sea. It is the purpose of this paper to provide further refinements in the scaling of $p$ over water surfaces.

The sea—atmosphere interfacial characteristics are commonly categorized in three classes: (i) smooth surface conditions; (ii) transition surface conditions; and (iii) rough surface conditions. Following Kondo (1975), for example, the first class is involved with wind speeds of less than 2–3 m s$^{-1}$ (at 10 m height), the second class is involved with a typical wind speed range of 3–8 m s$^{-1}$, and the third one with stronger wind speeds. In the bulk approximation for $p$ values, derived in section 2, these characteristics of the sea surface state, as well as refinements involved with computation of the potential temperature of the air at the top of the sublayer at the surface, are not considered. In section 3, however, these two aspects are introduced and the involved modifications of the bulk approximation for $p$ are evaluated.

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2. Methodology for bulk approximation of \( p \)

Sedecian (1980), based on a surface layer formulation, derived the following expression for \( p \):

\[
P = \frac{\phi_m(z/L)}{\ln(z/z_0) - \psi_m(z/L)},
\]

where the functional forms for \( \phi_m \) and \( \psi_m \) are taken from Businger et al. (1971). In (3) \( z_0 \) is the roughness length and the Monin-Obukhov length, \( L \), is defined as

\[
L = \frac{\bar{u}^2}{g k_0 \theta^*_e}.
\]

where \( \bar{\theta} \) is the typical potential temperature in the surface layer (300 K in this study); \( u \) and \( \theta \) are the friction velocity and surface layer buoyant temperature, respectively; \( g \) the gravity acceleration and \( k_0 \) is the Von Karman constant. The value of \( p \) is computed at height \( z \); however, it provides, as suggested by Sedecian, a good representation for a sublayer between \( z_1 \) and \( z_2 \) when \( z = (z_1 + z_2)^{0.5} \).

Based on (3), Sedecian constructed a nomogram for \( p \) as a function of \( z/L \) and \( z_0/L \) for typical conditions over land surfaces. Hence, according to this formulation, \( p \) can be determined at a given height \( z \), if \( L \) and \( z_0 \) are known. A similar methodology was used earlier by Panoñosky and Parsad (1965) to construct such nomograms. Unlike land surfaces, however, over water bodies, \( z_0 \) is dependent on the surface condition and the wind speed. Therefore, in addition to \( L \), \( z_0 \) also has to be computed, using the surface layer similarity formulations. Thus, it becomes somewhat cumbersome to use this nomogram over water surfaces, particularly if the purpose is for a straightforward simple evaluation of the wind profile, as is commonly needed in applied meteorology or engineering problems.

In the following sections two simplified and practical nomograms, which can be obtained through (3), for the specific case of the marine atmospheric surface layer, are presented. It will be shown that because of the characteristics of \( z_0 \) over water bodies, \( p \) values there can be obtained directly by knowing \( u \) and \( \theta \) at a single elevation, in addition to the sea surface temperature.

The most commonly used formulation for the value of \( z_0 \) over water bodies is based on Charnock (1955) who suggested the following relation:

\[
z_0 = \frac{\alpha \bar{u}^2}{g},
\]

where \( \alpha \) is a constant. The values for \( \alpha \) in (5) were evaluated in various studies (e.g., Clarke, 1970; Hicks, 1972; Sethuraman and Raynor, 1975, among others). Based on these studies, typically \( \alpha \) is in the range \( \sim 0.016 \) to \( \sim 0.064 \). In the current study we adopted relation (5) with \( \alpha = 0.032 \) (as recommended by Clarke, 1970), as a compromise value. Deviations from this value of \( \alpha \) have been shown to cause only small practical changes in the values of \( p \) (see section 3).

Next, a form for (3) based on direct observational data parameters will be considered. Since \( z/L \) is a function of the Richardson number, \( R_i \) (e.g., Businger et al., 1971), \( \phi_m \) and \( \psi_m \) in (3) are also functions of \( R_i \). Based on (1) they are also functions of the bulk Richardson number \( R_{iB} \) and \( p \).

The relation,

\[
\theta = k_0 \Delta \theta \left[ \ln \left( \frac{z}{z_0} \right) - \psi_R \left( \frac{z}{L} \right) \right],
\]

(6)

where \( \Delta \theta = \theta - \theta_0 \) is the difference between the potential temperature at height \( z \) and the potential temperature at height \( z_0 \), and \( \psi_R \) is the integrated profile function for the potential temperature, combined with (4) and (5) yields:

\[
\ln \left( \frac{z}{z_0} \right) = \ln \left( \frac{z}{L} \right) + \ln \left[ \frac{\bar{\theta} \cdot 0.74 \ln(z/z_0) - \psi_R(z/L)}{\alpha k_0 \Delta \theta} \right].
\]

(7)

The relation given in (7) implies that \( \ln(z/z_0) \) is a function of \( R_{iB} \) and \( \Delta \theta \). Based on (1) \( \ln(z/z_0) \) is also a function of \( R_{iB} \), \( p \) and \( \Delta \theta \). Consequently (3) provides a dependence of \( p \) on \( R_{iB} \) and \( \Delta \theta \). Since

\[
R_{iB} = z^2 \frac{\partial \theta}{\partial z} = \frac{z}{\bar{u}^2 \frac{\partial \theta}{\partial z}} \left[ \frac{z}{L} \right] \left[ \frac{\theta}{L} \right] \left[ \frac{z}{L^2} \Delta \theta \right],
\]

(8)

based on the previous discussion regarding the functional dependency of \( z/L \) and \( z_0/z_0 \). \( R_{iB} \) is a function of \( p, \bar{u}^2 \) and \( \Delta \theta \) which implies that for a given \( \Delta \theta \), the values of \( p \) calculated according to (3) over open water surfaces are a function of \( (z_0/L) \). It will be shown in the next section that replacing \( \theta_0 \) with \( \theta_0 \) (where \( \theta_0 \) is the sea surface skin potential temperature) in the definition of \( \Delta \theta \) has only a minor effect on the computed \( p \) values. Hence, in the specific case of sea surfaces, a simplified scaling of \( p \), by relating it directly to meteorological data and the sea surface temperature, is possible.

It should be noted that \( p \) values obtained in this derivation are limited to the layer in which \( u \), \( \theta \) and \( \theta_0 \) are observed. That layer is defined by the \( R_{iB} \) assumed in (8) using the observation height above the surface. Therefore, the \( p \) values derived under these relations would be best suited to derive wind speeds at levels below \( z \). On the other hand, this constraint does not exist when \( p \) is evaluated as a function of \( z_0 \) and \( L \) through (3) (since once \( z_0 \) and \( L \) are known, \( p \) can be computed at any \( z \) level within the surface layer). However, then, one has to compute \( z_0 \) and \( L \) in order to derive \( p \) values using this method.

Finally, over water bodies, because of the moisture effect on buoyancy, an evaluation of virtual temperature for use in \( \theta_0 \) may be of importance (e.g., Bianc, 1983). Following Liu et al. (1979), for example, therefore, \( \theta_0 \) can be replaced by \( \theta_0 = (1 - 0.61 \phi) \theta_0 \).
+ 0.61\theta \bar{q}_0$, where $v$ indicates virtual potential temperature; $q_0$ the specific humidity at level $z$ and $q_0$ the surface layer moisture scaling parameter. Using the Businger et al. (1971) relations for $\theta_0$ and $q_0$, and the approximation $\theta_0(\theta_0 \approx 1$, it can be shown that in order to consider the moisture-buoyancy effect in the previous formulations, $\Delta \theta$ should be replaced by: $\Delta \theta = \theta_{w} - \theta_{w} = \theta(1 + 0.61q_0) - \theta(1 + 0.61q_0)$. Unfortunatenly, $q$ profiles are not always measured over the sea in conjunction with $u$ and $\theta$ measurements. However, if they are available, or their characteristics can be prescribed, it is straightforward to consider $\Delta \theta_u$ rather than $\Delta \theta$.

Two types of nomograms were constructed in order to compute $p$ as outlined in sections 3 and 4.

3. Nomogram I

a. Bulk approximation of $p$

Equation (3) was solved for various feasible combinations of $u$, $\Delta \theta$, and $z$ (the range for $u$ was $1$ to $40$ m s$^{-1}$; for $\Delta \theta$ the range was $-10^\circ$ to $10^\circ$C, and for $z$ the range was $1$ to $30$ m; the adequacy of using the Businger et al. (1971) functional relations for this range of surface layer characteristics is discussed in section 6). The wind speed $u$ is at the height $z$, and $\Delta \theta$ corresponds to the difference in the air potential temperature between $z$ and $z_0$. Based on the output, we constructed the nomograms given in Figs. 1a and 1b, expressing the relation between $p$ and $z/\bar{u}^2$ for various values of $\Delta \theta$. Further refinements in this relation are evaluated in subsections 3b–d.

It was verified that for a given $\Delta \theta$, any combination of $z$ and $u$ which provides the same value of $z/\bar{u}^2$ results in the same value of $p$, as indeed expected from the conclusion in section 2. Additionally, tests were performed to evaluate the effect of possible uncertainties in $\alpha$ on the resultant $p$ values, using extreme values of $\alpha = 0.016$ and $0.064$. The ratio of $p$ values obtained with the lower $\alpha$ value as compared to those obtained with the higher one are presented in Table 1 (for selected values of $\Delta \theta$ and $z/\bar{u}^2$). The following characteristics are typical: (i) The ratio is closer to unity for the stable situations as compared to the unstable ones; and (ii) reduced ratio values are associated with intensification of the wind speed and low measurement elevation. Table 1 suggests, therefore, that for the intermediate value of $\alpha = 0.032$, the nomogram-computed $p$ values provide reasonable agreement with those derived for both of the extreme cases.

The nomograms in Figs. 1a and 1b indicate the following:

(i) For both unstable surface layers and stable surface layers situations, $p$ becomes independent of $\Delta \theta$ for

<table>
<thead>
<tr>
<th>$z/\bar{u}^2$</th>
<th>$\Delta \theta$ (°C)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$-10$</td>
</tr>
<tr>
<td>0.01</td>
<td>0.73</td>
</tr>
<tr>
<td>0.1</td>
<td>0.79</td>
</tr>
<tr>
<td>1</td>
<td>0.82</td>
</tr>
<tr>
<td>10</td>
<td>0.83</td>
</tr>
</tbody>
</table>
small values of \(z/u^2\) (for example, when the wind speed strengthens or \(z\) decreases, which causes \(Ri_a\) to tend towards its neutral value).

(ii) For both stability ranges, and for relatively large values of \(z/u^2\), the dependence of \(p\) on \(\Delta\theta\) for a given value of \(z/u^2\) is most pronounced close to the neutral conditions, while it reduces as \(\Delta\theta\) increases.

(iii) For relatively large \(z/u^2\), the dependence of \(p\) on \(\Delta\theta\) is much more pronounced for the stable situations, as compared to the unstable situations.

(iv) The range of \(p\) values in the unstable region is typically between 0.02 to 0.2, while for the stable case the range is 0.1 to \(\sim 1.0\). (A very large change of the wind speed with height associated with a very stable layer is indicated by \(p \approx 1\).)

While the presented nomogram provides a general characterization for wind profiles in the marine surface layer, its application is not straightforward because of the need to evaluate \(\theta_{20}\). It would become more useful if the sea surface potential temperature, \(\theta_s\), can be considered, rather than \(\theta_{20}\). Such a replacement is possible when introducing a refined lower boundary condition for the profile of \(\theta\) in which the interfacial laminar layer is considered. (See, for example, Kondo, 1975; Liu et al., 1979 for a detailed formulation and discussion related to the introduction of these boundary conditions.) To evaluate this alternative, we used Owen and Thomson's (1963) and Kondo's (1975) derivations for the interfacial marine sublayer. Accordingly, \(\theta_{20}\) used in Eq. (6) is given as

\[
\theta_{20} = \theta_s + B_{H^{-1}} \theta_s
\]

where \(B_{H^{-1}}\) is the reciprocal of the Stanton number. The Stanton number is a function of the roughness Reynolds number and the Prandtl number. It expresses the fact that the heat transfer capacity of the air away from the water surface is increased by the roughness generated turbulence. However, the heat transfer rate from the surface is constrained by the magnitude of the air molecular heat conduction. The expression for the Stanton number used in the present study is given in subsection 3b. In subsections 3b, c and d, the effect of this refinement on results obtained with the bulk approximation for \(p\) is evaluated for the three states of the sea surface.

b. Rough sea surface conditions

For rough sea surface condition, following Owen and Thomson (1963, p. 332) Kondo (1975, p. 94):

\[
B_{H^{-1}} = 0.54 \left( \frac{u_* 15z_0}{\nu} \right)^{0.45},
\]

where \(\nu = 0.15 \text{ cm}^2 \text{ sec}^{-1}\) is the kinematic viscosity of air.

Considering (5), (9) and (10), it can be seen that the correction term in (9) is proportional to \(u_*^{0.35}(u_* \theta_s)\). When \(u\) is in the lower range of wind speeds related to rough sea conditions (i.e., \(\sim 8 \text{ m sec}^{-1}\)) then for a given \(z\), the corresponding \(z/u^2\) value is largest for rough sea condition. According to nomograms in Kondo (1975) in that situation, \(u_*\) and \((u_* \theta_s)\) are the lowest for rough sea conditions, and therefore \(\theta_s\) and \(\theta_{20}\) tend to be relatively close. Similarly, when the surface layer tends to become neutral (i.e., \(\theta_s \rightarrow 0\)), then the correction term in (9) diminishes; thus, \(\theta_s \approx \theta_{20}\). On the other hand, as \(u\) increases (i.e., \(z/u^2\) decreases), the differences between \(\theta_{20}\) and \(\theta_s\) can increase significantly. However, in the computation of \(p\), as evident from Fig. 1, the differences in \(p\) with variations of \(\Delta\theta\) for relatively small values of \(z/u^2\) are generally minor when large values of \(\Delta\theta\) occur. We evaluated quantitatively the significance of replacing \(\theta_{20}\) by \(\theta_s\) in the definition of \(\Delta\theta\) for the computation of \(p\). For this purpose we substituted \(\theta_{20}\) as given in (9) in (6) and computed \(p\) and \(\theta_s - \theta_{20}\) for the combinations of values of \(z\), \(u\) and \(\theta_s - \theta_s\). It was found that the new computed \(p\) values differ, at most, by 10% from those given in Fig. 1. This result can be easily verified with the help of Table 2; for a given value of \(z/u^2\), evaluate the difference in \(p\) obtained in the nomograms while using \(\theta_s - \theta_{20}\) as compared with the corresponding \(p\) value while using \(\theta_s - \theta_{20}\).

| Table 2. The computed \(\Delta\theta = \theta_s - \theta_{20} (\text{°C})\) for selected values of \(z/u^2\) and \(\theta_s - \theta_s\) when the relation given in (9) is considered. |
|---|---|---|---|---|---|---|---|---|---|---|
| \(z (\text{m})\) | \(z/u^2\) | -10 | -8 | -6 | -4 | -2 | 0 | 2 | 4 | 6 | 8 | 10 |
| 1 | 0.0156 | -7.46 | -5.97 | -4.48 | -2.99 | -1.44 | 0 | 1.59 | 3.20 | 4.82 | 6.45 | 8.09 |
| 0.0100 | -6.36 | -5.10 | -4.12 | -2.55 | -1.27 | 0 | 1.41 | 2.83 | 4.25 | 5.69 | 7.13 |
| 10 | 0.1563 | -8.65 | -6.93 | -5.20 | -3.48 | -1.74 | 0 | 1.82 | 3.70 | 5.60 | 7.59 | 9.59 |
| 0.0104 | -2.81 | -2.25 | -1.69 | -1.13 | -0.56 | 0 | 0.69 | 1.38 | 2.08 | 2.78 | 3.50 |
| 20 | 0.3125 | -8.82 | -7.07 | -5.31 | -3.55 | -1.28 | 0 | 1.87 | 3.83 | 5.84 | 7.87 | 9.10 |
| 0.0125 | -2.34 | -1.87 | -1.40 | -0.94 | -0.47 | 0 | 0.57 | 1.16 | 1.74 | 2.33 | 2.93 |
| 30 | 0.4688 | -8.91 | -7.14 | -5.36 | -3.58 | -1.80 | 0 | 1.90 | 3.90 | 5.93 | 7.96 | 9.49 |
| 0.0197 | -2.80 | -2.24 | -1.68 | -1.12 | -0.56 | 0 | 0.67 | 1.35 | 2.05 | 2.74 | 3.45 |
c. Smooth sea surface conditions

In this state of the sea surface, $z_0$ is evaluated through the relation (e.g., Kondo, 1975; Liu et al., 1979):

$$z_0 = \frac{0.11 \nu}{u_*}$$  \hspace{1cm} (11)

Following Kondo, the reciprocal Stanton number for this state of the sea is given as a constant, i.e., $B_n^{-1} = -2.7$.

The modification of the relation between $z_0$ and $u_*$ as compared to that given in (5) does not permit $p$ to be expressed through the simplified functional dependence suggested in section 2. However, equating (5) and (11) shows that for $u_* \approx 8$ cm s$^{-1}$, $z_0$ computed by both formulations are identical. Since for a smooth sea surface, it is likely that $u_* \approx 10$ cm s$^{-1}$, the following differences in the computed $p$ values are assumed to be obtained by replacing Eq. (5) with Eq. (11):

(i) Differences in computed $p$ are likely to increase with the reduction of $u_*$ (i.e., with light winds).

(ii) The differences are likely to increase as the thermal stratification of the surface layer tends toward the neutral, since the logarithmic term in (3) involved with $z_0$ then tends to become dominant. On the other hand, for diabatic surface layers, particularly those that are stable, the weight of the logarithmic term in the computation of $p$ is of less importance.

The symbols in Fig. 1 illustrate the $p$ values computed using (11) and the Stanton number involved with smooth sea surface conditions in (9) for various combinations of $\theta_e - \theta_n$ and $u_*$ ($u_* \approx 3$ m s$^{-1}$). For the neutral and the unstable surface layers the largest deviations of $p$ from the bulk approximation values are related to light wind speeds ($u_* \approx 1$ m s$^{-1}$). Stable surface layer values show excellent agreement with those computed with bulk approximations of $p$. Thus, it is suggested that computed $p$ values based on a refinement to smooth sea surface conditions are in close agreement with those derived with the bulk approximation (as presented in subsection 3a).

d. Transition sea surface conditions

This sea state is practically involved with an intermediate interfacial characteristic similar to that typical in 3b and 3c. Therefore based on the evaluations in 3b and 3c, interpolation of the $p$ values for the transition sea conditions can be obtained.

4. Nomogram II

In this section a nomogram of $p$ as dependent on $z/z_0$ and $z/L$ similar to Sedefan (1980) is derived (nomogram II). However, in the current derivation, a range of $z/z_0$ which is typical for water surfaces—rather than typical to land surfaces as in the Sedefan study—is considered (the range considered for $z/z_0$ is $10^2$-$10^6$). Nomogram II is presented in Fig. 2, showing a range of $p$ values obtained in Nomogram I. From the mathematical–physical point of view both nomograms are equivalent as to the derived $p$ values. As stated in previous sections the present nomogram has the advantage of providing the dependency of $p$ on $z$. However, it has the disadvantage of being unrelated directly to measured meteorological parameters as in Nomogram I. In order to enable using Nomogram II in a direct way, Nomogram III was constructed to compute the values of $z/L$ as dependent on $z/u^2$ and $\Delta \theta$ (Fig. 3). When $u$ at level $z$ and the corresponding $\Delta \theta$ values are given, the value of $z/L$ can be obtained from Nomogram III, while the corresponding value of $p$ at level $z$ can be obtained from Nomogram I. Since $p$ values obtained by Nomograms I and II are identical at level $z$, based on the $p$ value obtained from Nomogram I, and the corresponding $z/L$ value obtained from Nomogram III, the corresponding value of $z/z_0$ can then be obtained from Nomogram II. Once the values of $z/L$ and $z/z_0$ are given at a specific elevation, $z$, their values are available for any other elevation (assuming that $L$ is constant within the surface layer). Therefore, using this procedure, a single elevation value of $u$ and the corresponding value of $\Delta \theta$ can be used through Nomograms I and III to establish $p$ values at any given height $z$ from Nomogram II. Finally, evaluation of the appropriate range of $z/L$ for which the Businger et al. (1971) formulation provides an accurate representation for the surface layer velocity is given in section 6.

5. Nomograms validations

In order to validate the accuracy of computed $p$ values based on the previous nomograms, data from three offshore measurement sites were used. Table 3 summarizes several pertinent characteristics of the observed data. The data include wind speed, air and sea surface temperatures, and air specific humidities in most cases. Some of the data taken from Donelan et al. (1975) reflecting very strong wind situations were, however, without moisture profiles. For those cases it was assumed $q_e = q_e - \delta g \text{ kg}^{-1}$. The value of $\Delta \theta$ was computed with $\theta_n$ at the sea surface determined assuming a saturated specific humidity. The data includes both stable and unstable surface layer situations. Winds measured at levels $z_1$ and $z_2$ (Table 3) were used to compute observed $p$ values based on (2). The nomogram-derived $p$ values were evaluated based on observed values of $\theta_e$ and $u$ at the level $z_1 \sim (z_1 z_2)^{1/2}$ and $\theta_n$ at the sea surface. Observed $p$ values were compared to values derived based on Nomograms I and II. The comparison features were identical for both nomograms cases; therefore, results obtained for comparison based on Nomogram I are presented (Fig. 4). With the unstable surface layer (Fig. 4a), most deviations from observed values were in the range $\pm 0.02$. With the sta-
Fig. 2. The relation between $p, z/L$ and $z/z_0$ over water surfaces with (a) unstable surface layer, and (b) stable surface layer (Nomogram II).

6. Conclusions

The values of $p$ to be used in the extrapolation of wind profiles in the marine surface layer were evaluated through a bulk approximation approach which per-

Fig. 3. The relation between $z/L$ and $z/u^2$ for various values of $\Delta \theta$ (in °C) as labeled on the plotted curves (Nomogram III).
mitted the construction of simplified nomograms. Refinements to this approach including consideration of surface sea temperature and state of the sea suggest that the bulk approximation generally provides accurate $p$ values. The range of $p$ values in the unstable surface layer is between 0.02–0.2, while for the stable situations the range is 0.1–1.0. The values of $p$ converge to about 0.2 for high wind speeds.

In order to evaluate $p$ by these nomograms, measured values of $u$ and $\theta$ are needed at one height $z$ within the surface layer in addition to the sea surface temperature, $\theta_s$. Evaluations for $\theta_s - \theta_a$ can be reasonably accurate for practical applications when the vertical gradient of $\theta$ is sufficiently large. Since generally $\Delta T$ is measured, the relation $\Delta \theta = \Delta T + \gamma_d \frac{z}{L}$ can be used while applying the nomogram, where $\gamma_d$ is the dry adiabatic lapse rate.

Improvements of the performance of the nomograms is obtained when virtual potential temperature effects for $\theta_v - \theta_a$ (i.e., to include the effect of moisture on buoyancy) are considered. At the sea surface, virtual temperature can easily be obtained by assuming saturated air conditions. Thus, if moisture information is known at level $z$, the modification in $\theta_v - \theta_a$ can be estimated easily as outlined in the paper.

A major point to be considered while using the nomograms presented in this paper is the range of $z/L$ in which the functional relations given in Businger et al. (1971) (which were adopted in the present study) provide an accurate representation of the velocity profile within the surface layer. For the unstable surface layer this formulation is very accurate, at least for $0 < z/L \approx -2$ (Panofsky and Dutton, 1983); it is likely to have increasing inaccuracies as $z/L$ values decrease below -2. However, Roberto et al. (1985) found this formulation to perform well within the range $-8 < z/L \leq 0$. For the stable surface layer, Skibin and Businger (1985), reviewing previous studies and also analyzing additional data, concluded that the formulation is accurately represented at least up to $z/L \approx 1$. The range of $z/L$ values adopted in the construction of the nomograms in the present study is relatively wide. However, while using the nomograms, one has to consider the possible reduction in accuracy with large values of $|z/L|$. Based on the meteorological measurements needed to determine the $p$ values and using Nomogram

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**Table 3. General information about the measurements used for the validation of the nomograms.**

For definitions of $z_1$, $z_2$, $z_3$, see section 5.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Location</th>
<th>Measurement platform</th>
<th>Range of wind speed (m s$^{-1}$)</th>
<th>$z_1$</th>
<th>$z_2$</th>
<th>$z_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Donelan et al. (1975)</td>
<td>Lake Ontario; coastal water</td>
<td>offshore tower</td>
<td>1-17</td>
<td>2.59</td>
<td>12.57</td>
<td>5.61</td>
</tr>
<tr>
<td>Fleagle et al. (1958)</td>
<td>East Sound; inland water</td>
<td>raft</td>
<td>3-9</td>
<td>0.31</td>
<td>4.42</td>
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</tr>
<tr>
<td>Badgley et al. (1972)</td>
<td>Indian Ocean; open and coastal water</td>
<td>buoy</td>
<td>2-8</td>
<td>1.57</td>
<td>8.15</td>
<td>2.97</td>
</tr>
</tbody>
</table>

**Fig. 4. Observational evaluation of Nomogram I (a) unstable surface layer, and (b) stable surface layer.**

The plus marks on the symbols reflect cases in which moisture data were unavailable.
III, the related z/L values can be established, and consequently the relative accuracy of the derived p values. Finally, validation of the presented nomogram based on observational data suggests reasonable accuracy.

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