Note: The Relationship Between Numerical and Physical Models of Atmospheric Flow

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1 Introduction[1]

There are two fundamental methods of simulating atmospheric flows - *physical models* and *mathematical models*. With the first technique, scale model replicas of observed ground surface characteristics (e.g., topographic relief, buildings) are constructed and inserted into a chamber such as a wind tunnel or a water tank. The flow of air or other gases or liquids in this chamber is adjusted so as to best represent the larger scale, observed atmospheric conditions. Mathematical modeling, by contrast, utilizes such basic analysis techniques as algebra and calculus to solve directly the conservation laws of motion, heat, moisture, and other atmospheric constituents.

2 Physical models

Using order of magnitude estimates for the dependent variables and assuming that L and S are the representative length and velocity scale of the circulation of interest (i.e., $\frac{S^2}{L}$ = maximum of $\left[\frac{W^2}{L_z}, \frac{WU}{L_z}, \frac{WU}{L_x}, \frac{U^2}{L_x}\right]$), then a scaled version of the conservation of motion equation can be written as

$$\begin{aligned}\left[\frac{S^2}{L}\right]\frac{\partial\hat{\bar{u}}_i}{\partial\hat{t}} = & -\left[\frac{S^2}{L}\right]\hat{\bar{u}}_j\frac{\partial\hat{\bar{u}}_i}{\partial\hat{x}_j} - \left[\frac{e_{u_i}^2}{L}\right]\frac{\partial}{\partial\hat{x}_j}\widehat{\overline{u_j''u_i''}} \\ & -\left[\frac{R\delta\theta}{L}\right]\hat{\theta}_0\frac{\partial\hat{\pi}'}{\partial\hat{x}_i} - \left[\frac{R\delta\theta}{L}\right]\hat{\theta}_0\left\{\frac{\partial\hat{\pi}_0}{\partial\hat{x}}\delta_{i1} + \frac{\partial\hat{\pi}_0}{\partial\hat{y}}\delta_{i2}\right\} \qquad (1) \\ & +\left[\frac{\delta\theta}{\theta_0}g\right]\hat{\theta}'\delta_{i3} - [\Omega S]\,2\varepsilon_{ijk}\hat{\Omega}_j\hat{\bar{u}}_k - \left[\frac{\nu S}{L^2}\right]\frac{\partial^2\hat{\bar{u}}_i}{\partial\hat{x}_j\partial\hat{x}_j}\end{aligned}$$

where a circumflex (ˆ) over a dependent or independent variable indicates that it is nondimensional. The scaling parameter e_{u_i} is a measure of the subgrid scale velocity correlations that can be estimated from the mean subgrid scale kinetic energy i.e.,

$$e_{u_i} = [\overline{u_i''^2}/2)]^{1/2}.$$

Including an estimate for the molecular viscous dissipation and multiplying (1) by L/S^2 (to obtain a nondimensional equation for the local acceleration) results in

$$\begin{aligned}\frac{\partial\hat{\bar{u}}_i}{\partial\hat{t}} = & -\hat{\bar{u}}_j\frac{\partial\hat{\bar{u}}_i}{\partial\hat{x}_j} - \left[\frac{e_{u_i}^2}{S^2}\right]\frac{\partial}{\partial\hat{x}_j}\widehat{\overline{u_j''u_i''}} - \left[\frac{R\delta\theta}{S^2}\right]\hat{\theta}_0\frac{\partial\hat{\pi}'}{\partial\hat{x}_i} \\ & -\left[\frac{R\delta\theta}{S^2}\right]\hat{\theta}_0\left\{\frac{\partial\hat{\pi}_0}{\partial\hat{x}}\delta_{i1} + \frac{\partial\hat{\pi}_0}{\partial\hat{y}}\delta_{i2}\right\} + \left[\frac{gL\delta\theta}{\theta_0 S^2}\right]\hat{\theta}'\delta_{i3} \qquad (2) \\ & -\left[\frac{\Omega L}{S}\right]2\varepsilon_{ijk}\hat{\Omega}_j\hat{\bar{u}}_k - \left[\frac{\nu}{LS}\right]\frac{\partial^2\hat{\bar{u}}_i}{\partial\hat{x}_j\partial\hat{x}_j}\end{aligned}$$

To use a scaled physical model to represent accurately the conservation-of-motion relation in the atmosphere, it is essential that

1. the individual bracketed terms be equal in the model and in the atmosphere, or
2. the bracketed terms that are not equal must be much less in magnitude than the other bracketed terms in Eq. (2).

When these conditions are met the actual and modeled atmospheres are said to have *dynamical similarity*.

Two of these bracketed terms are defined as

$$\Omega L/S = 1/R_o$$

and

$$v/LS = 1/Re,$$

where R_o is the Rossby number and Re the Reynolds number; and

$$gL\,(\delta\theta/\theta_o)\,/S^2 = \mathrm{Ri}_b$$

is the *bulk Richardson number*. ($\delta\theta$ represents the potential temperature perturbation and is the same order as the temperature perturbation δT).

From Eq. (2), to maintain dynamic similarity, it is implied that to represent all of the terms in the equation properly:

1. the ratio of the subgrid scale kinetic energy to the grid-volume average kinetic energy must be kept constant;
2. reducing the length scale L in the physical model requires:
   (a) an increase in the magnitude of the horizontal temperature perturbation $\delta\theta$ or a reduction in the simulated wind flow speed S or both,
   (b) an increase in the rotation rate Ω or a reduction in S or both,
   (c) a decrease in the viscosity ν or an increase in S or both;
3. an increase in $\delta\theta$ in the pressure gradient term necessitates that S also increase.

Unfortunately, it is impossible to satisfy all of these requirement simultaneously in existing physical models of mesoscale atmospheric circulations. Such physical models are constructed inside of buildings, which limits the dimensions of the simulated circulations to the size of meters, whereas actual mesoscale circulations extend over kilometers.

To illustrate the difficulty of obtaining dynamic similarity in a physical model for all terms in Eq. (2), let the horizontal scale of a mountain ridge be 10 km, whereas the physical model of this geographic feature utilizes a 1 m representation. The scale reduction is, therefore, 10^4. Thus, if $S = 10$ m s^{-1} in the real situation and air is used in the scaled model atmosphere, then the simulated wind speed would have to be 10^5 m s^{-1} to maintain identical Reynolds number similarity! In addition, to have the same Rossby number for this example, the physical model must rotate 10,000 times more rapidly than the earth or the wind speed must be reduced by 10,000. Reducing the speed, of course, is contradictory to what is required to obtain Reynolds number similarity! Only if the results are relatively insensitive to changes in these nondimensional quantities, as suggested, for example, by Cermak (1975) for large values of the Reynolds number in simulations of the atmospheric boundary layer, can one ignore large differences in the nondimensional parameters.

[1]This note was extracted and modified somewhat from that presented in Pielke (1984, Chapter 5).

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From the example just given, however, it should be clear that it is impossible to obtain *exact* dynamic similarity between mesoscale atmospheric features and the physical model when all of the terms in Eq. (2) are included.

Using the same assumptions applied to produce Eq. (1), the conservation-of-heat relation, represented by the potential temperature equation, can be written as

$$\left[\frac{\delta\theta S}{L}\right]\frac{\partial\hat{\bar{\theta}}}{\partial\hat{t}} = -\left[\frac{\delta\theta S}{L}\right]\hat{\bar{u}}_j\frac{\partial\hat{\bar{\theta}}}{\partial\hat{x}_j} - \left[\frac{e_\theta e_{u_i}}{L}\right]\frac{\partial}{\partial\hat{x}_j}\overline{\hat{u}_j''\hat{\theta}''} + \left[\frac{\delta\theta S}{L}\right]\hat{\bar{S}}_\theta, \quad (3)$$

where $e_\theta e_{u_i}$ is a measure of the subgrid scale correlation between the fluctuating velocities and temperatures, with e_θ perhaps represented by

$$e_\theta = [(\overline{\theta''^2}/2)]^{1/2}.$$

If the molecular conduction of potential temperature C_θ is included in (3) and represented in analogy with the viscous dissipation term as

$$C_\theta = \frac{k_\theta}{\rho C_p}\frac{\partial^2\bar{\theta}}{\partial x_j \partial x_j}; \quad |C_\theta| \sim \frac{k_\theta}{\rho_0 C_p}\frac{\delta\theta}{L^2},$$

where k_θ is the potential temperature molecular conduction coefficient, then multiplying by $L/\delta\theta S$ and including the order of magnitude estimate of C_θ yields

$$\frac{\partial\hat{\bar{\theta}}}{\partial\hat{t}} = -\hat{\bar{u}}_j\frac{\partial\hat{\bar{\theta}}}{\partial\hat{x}_j} - \left[\frac{e_\theta e_u}{\delta\theta S}\right]\frac{\partial}{\partial\hat{x}_j}\overline{\hat{u}_j''\hat{\theta}''} + \left[\frac{k_\theta}{\rho_0 C_p v}\right]\left[\frac{\nu}{LS}\right]\frac{\partial^2\hat{\bar{\theta}}}{\partial\hat{x}_j^2} + \hat{\bar{S}}_\theta. \quad (4)$$

The ratio

$$k_\theta/\rho_0 C_p \nu = Pr^{-1}$$

is called the *Prandtl number* and is of order unity for air.

Thus to obtain *thermal similarity* between the mesoscale circulation and its laboratory representation, the Reynolds number must also be very large and the partitioning of heat transport between the subgrid scale and resolvable fluxes must be the same. If in Eq. (2), for example, the temperature perturbation $\delta\theta$ must be increased in the bulk Richardson number Ri_b to compensate for a decrease of L in the laboratory model, then in Eq. (4) the turbulent fluctuations in the simulated atmosphere must also be increased.

The nondimensional source-sink term for potential temperature $\hat{S}_\theta$ is included in the analysis. However, the mathematical procedure of representing it as a single variable masks its physical complexity. This term includes such effects as radiative flux divergence, phase changes of water, etc. and is an involved function of the dependent variables. Thus it is extremely difficult to evaluate this term using scale analysis, and, in practice, physical modelers exclude it in their representation of atmospheric flows. An equivalent similarity analysis can be performed for water substance and other aerosol and gaseous contaminants. Because of the inability to accurately represent the sources and sinks of these variables, (i.e., $\hat{\bar{S}}_{q_n}$ and $\hat{\bar{S}}_{\chi_m}$), however, physical modelers have only studied the movement of nonreactive, conservative pollutants around terrain and building obstacles.

In utilizing physical models, the conservation-of-mass relation must also be satisfied, and, as long as the ratio of the variations of specific volume to the average specific volume in the physical model are much less than unity explicit temporal changes in density can be ignored in this conservation relation. The scaled version of the incompressible conservation-of-mass equation, $\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$, shows that

$$W \sim L_z U/L_x, \quad W \sim L_z V/L_y$$

so that, if the ratio of the vertical to the horizontal scales of the circulation is kept constant between the physical model and the atmosphere, *kinematic similarity* is obtained. This requirement could be satisfied providing the horizontal to vertical representation of the terrain and other physical features of the ground surface in the physical model are not exaggerated. This latter condition is called *geometric similarity*.

The final similarity conditions needed in physical models include the requirement that air flowing into the simulated region have velocity and temperature profiles scaled according to the nondimensional relations given by Eqs. (2) and (4) and that the flow will be close to equilibrium (i.e., $\partial\hat{\bar{u}}_i/\partial\hat{t}$ and $\partial\hat{\theta}/\partial\hat{t}$ are small, relative to the remaining terms in Eqs. (2) and (4)). In addition, such bottom conditions as surface temperature and aerodynamic roughness must be scaled so as to produce kinematic, dynamic, and thermal similarity in the lowest levels of the physical model. These requirements are referred to as *boundary similarity* and their creation necessitates a comparatively long fetch from the input region of the laboratory apparatus to the region of simulation, as well as obstacles such as a lattice placed upwind in the flow to generate specific velocity profiles and turbulence characteristics.

With all of these requirements, physical modeling of the mesoscale has been primarily limited to stably stratified flows over irregular terrain. Even for this case, however, such observed features of the real atmosphere as the veering of the winds with height, radiational cooling, and condensation cannot be reproduced.

3 Use of numerical models to provide boundary conditions for the physical models

Pielke (1984) reviews in depth the techniques of numerical modeling of atmospheric flows. For the purpose of this paper, such numerical models can be used to remove some of the constraints placed on physical models by providing:

1). the appropriately scaled velocity, temperature, moisture, and other atmospheric constituent profiles in the inflow stream to the physical model;

and

2). the appropriate surface fluxes of heat, moisture and momentum, as scaled for the physical model (e.g., $e_{u_i}^2/S^2$ from (2), and $e_\theta e_u/\delta\theta S$ from (4)).

In this approach, the numerical-physical models are linked in a one-way nested grid approach where the numerical atmospheric model provides a realistic flow simulation for a larger scale domain at a scale where the physical model suffers from similarity limitations. This time-evolving flow information is used as boundary conditions for the smaller domain physical model. The physical model can provide realistic simulations on the smaller scale where the numerical model suffers from computational limitations due to the small domain size and resultant large sensitivity of the results to even slight errors in the lateral boundary conditions of the numerical model (Pielke, 1987). In addition, the numerical models fail to adequately resolve nearly discontinuous obstacles such as cliffs, buildings, individual trees, etc.

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5 References

[1] Cermak, J.E., (1975): Applications of fluid mechanics to wind engineering - a Freeman Scholar Lecture. *J. Fluids Eng.*, **97**, 9-38.

[2] Pielke, R.A., (1984): *Mesoscale Meteorological Modeling.* Academic Press, New York, N.Y., 612 pp.

[3] Pielke, R.A., (1987): The challenge of using mesoscale data in mesoscale models. *Proceedings of the Symposium on Mesoscale Analysis and Forecasting, Incorporating Nowcasting*, August 17-19, 1987, Vancouver, British Columbia, Canada, ESA, 651-652.

List of Symbols

Dependent Variables

u_i = velocity vector

θ = potential temperature

π = scaled pressure; $\pi = C_p(p/1000 \text{ mb})^{R/C_p}$ where C_p is the specific heat at constant pressure, p is pressure in millibars

Constants

R = gas constant

Ω = rotation rate of the earth in radians per second

ν = kinematic viscosity of air

g = gravitational acceleration

k_θ = molecular coefficient of conduction of potential temperature

Mathematical Operations

$(\bar{\ })$ = grid-volume average of dependent variable, e.g., $\bar{u}_i, \bar{\theta}$, etc.

$(\ )'$ = resolvable perturbation of dependent variable from domain-average variable

$(\ )_0$ = domain average

$(\hat{\ })$ = nondimensional variable

$(\ )''$ = subgrid scale perturbation of dependent variable from grid-volume average

Scaling Values

L = length scale

$\delta\theta$ = scaled temperature gradient

S = scaled velocity

Independent Variable

x_i = spatial independent variable

t = time