Comments on "An Analysis of Closures for Pressure-Scalar Covariances in the Convective Boundary Layer"

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29 October 1987 and 7 January 1988

Moeng and Wyngaard (1986) decompose the fluctuating pressure field in a convective boundary layer into several components, each related via the Poisson equation to a term in the prognostic equation for velocity. In a digression from the main subject of the paper they consider the effect of spatial fluctuations in the surface temperature on the buoyancy component, p_B. Assuming a sinusoidal surface temperature variation of amplitude 5 K and wavelength 5 km, they estimate near-surface pressure fluctuations on the order of 10^2 Pa, which is 25 times larger than the pressure fluctuations observed by Gal-Chen and Kropfli (1984) and also much larger than pressure fluctuations calculated by Moeng and Wyngaard's large-eddy simulations. In this comment we review their analysis and conclude that it does not take proper account of the temperature gradient near the surface. We also review briefly some evidence concerning the magnitude of fluctuations in temperature—at the surface and immediately above it—produced by variations in land use on scales of a few kilometers. Using a modified form of Moeng and Wyngaard's argument we estimate that near-surface fluctuations in p_B produced directly by these variations will be on the order of 10^0 Pa.

As defined by Moeng and Wyngaard, p_B is governed by the equation,

$$\frac{1}{\rho_0}\nabla^2 p_B = \frac{\partial}{\partial z}\beta g\theta, \tag{1}$$

with upper and lower boundary conditions

$$\left.\frac{\partial p_B}{\partial z}\right|_0 = \rho_0\beta g\theta_S \quad \left.\frac{\partial p_B}{\partial z}\right|_{z_T} = 0, \tag{2}$$

where θ_S is the fluctuating temperature at the surface. They then divide p_B into two parts, $p_B{}^{(1)}$ and $p_B{}^{(2)}$, such that

$$\frac{1}{\rho_0}\nabla^2 p_B{}^{(i)} = \frac{\partial}{\partial z}\beta g\theta^{(i)} \tag{3}$$

$$\left.\frac{\partial p_B{}^{(i)}}{\partial z}\right|_0 = \rho_0\beta g\theta^{(i)}|_0 \quad \left.\frac{\partial p_B{}^{(i)}}{\partial z}\right|_{z_T} = 0 \tag{4}$$

for $i = 1$ and 2, where

$$\theta^{(1)} = \begin{cases}\theta & z > 0\\ 0 & z = 0\end{cases} \tag{5}$$

$$\theta^{(2)} = \begin{cases}0 & z > 0\\ \theta_S & z = 0.\end{cases} \tag{6}$$

(Moeng and Wyngaard do not use the symbols $\theta^{(1)}$ and $\theta^{(2)}$, but we find it convenient to introduce them.) Note that $\theta^{(2)}$ has a discontinuity at $z = 0$, and if θ is continuous then $\theta^{(1)}$ also has a discontinuity of equal magnitude and opposite sign. Assuming a sinusoidally fluctuating surface temperature,

$$\theta_S = \Theta_S e^{i(k_x x + k_y y)}, \tag{7}$$

their solution of (3) and (4) for $p_B{}^{(2)}$ in the limit $z_T \to \infty$ is

$$p_B{}^{(2)} = -\frac{\rho_0\beta g\Theta_S}{k}e^{-kz}e^{i(k_x x + k_y y)}, \tag{8}$$

where $k^2 = k_x{}^2 + k_y{}^2$. (A typographical error in Moeng and Wyngaard's paper has been corrected in the above equation.) Substitution of $\Theta_S = 5$ K and $2\pi/k = 5$ km yields an amplitude for $p_B{}^{(2)}$ at the surface of about 10^2 Pa. The following paragraph is reproduced from Moeng and Wyngaard (1986):

> We interpret this result as follows: surface temperature fluctuations generate large near-surface pressure perturbations, and the resulting local pressure gradients cause changes in the velocity and temperature fields in the overlying flow. These changes might modify the lower boundary condition [on p], enabling the friction near the lower surface to balance some of the imposed buoyancy and reducing the magnitude of p_B. In addition, the pressure fluctuations associated with the changes in velocity and temperature could be nega-

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tively correlated with $p_B^{(2)}$, reducing the pressure variance to the observed levels.

There is no discussion of $\theta^{(1)}$ or $p_B^{(1)}$. As we understand it, they are assuming, in first approximation, that the fluctuating temperature is large at the surface and small elsewhere, and so can be represented by $\theta^{(2)}$. We present evidence below that supports this approximation. We contend, however, that they have not integrated the Poisson equation for $p_B^{(2)}$ correctly across the discontinuity in $\theta^{(2)}$.

We propose a simple modification to Moeng and Wyngaard's approach. We let $\theta^{(2)} = \theta_S$ at the surface, but have it decrease rapidly but continuously to zero at some small height z_S. For example, let

$$\theta^{(2)}(x, y, z) = \Theta(z)e^{i(k_x x + k_y y)} \tag{9}$$

where $\Theta(z)$ is continuous and has value Θ_S at $z = 0$. The solution for $p_B^{(2)}$ is then of the form

$$p_B^{(2)} = P(z)e^{i(k_x x + k_y y)} \tag{10}$$

where

$$\frac{1}{\rho_0}\left(\frac{\partial^2 P}{\partial z^2} - k^2 P\right) = \frac{\partial}{\partial z}\beta g \Theta \tag{11}$$

$$\left.\frac{\partial P}{\partial z}\right|_0 = \rho_0 \beta g \Theta_S \quad \left.\frac{\partial P}{\partial z}\right|_{z_T} = 0. \tag{12}$$

Now, $\partial\Theta/\partial z \sim -\Theta_S/z_S$ below $z = z_S$, so for $kz_S \ll 1$ we neglect the $-k^2P$ term there in seeking the solution to (11). (The alternative is to have P near the surface very large, $\sim -\rho_0\beta g\Theta_S/z_S k^2$, but this leads to a contradiction.) Integrating up from the surface gives

$$\frac{\partial P}{\partial z} = \rho_0 \beta g \Theta \quad \text{for} \quad 0 \leqslant z \leqslant z_S. \tag{13}$$

This provides a lower boundary condition on P for the region between $z = z_S$ and $z = z_T$ and the solution of (11) there is just $P = 0$. The amplitude of the surface pressure fluctuation can now be found by integrating (13) down from z_S:

$$P(0) = -\rho_0 \beta g \int_0^{z_S} \Theta dz. \tag{14}$$

As $z_S \rightarrow 0$, this amplitude goes to zero. We believe that this is the correct solution for the limit which Moeng and Wyngaard consider.

There is observational evidence that a surface temperature variation of amplitude ~ 5 K or more is not uncommon on a strongly heated, heterogeneous land surface (Lenschow and Dutton 1964). However, given the strong lapse rate that is observed near the surface in these conditions and the likelihood that the lapse rate will be greater over a warmer surface, one expects that the air temperature variation will be much smaller at heights more than a few meters above the ground. For example, Businger and Frisch (1972) report surface radiation temperature deviations of -20 K, relative to the surrounding land, for horizontal distances up to 1 km over irrigated fields and ponds in Kansas, but their traces of air temperature at a height of 30 m are not perturbed by more than 1 K by these features. Holmes (1969) observed the surface radiation temperature and air temperature over Lake Pakowki, an irregularly shaped shallow lake in Alberta with a maximum dimension of 20 km. In the middle of a typical summer day (3 August 1967) the lake had a surface temperature approximately 25 K cooler than nearby prairie, but the maximum cooling of the air near the downwind shore was only 3 K at a height of 15 m and unmeasurable at 75 m. Over an irrigation project of area 12 km^2 Holmes found the surface temperature was 8 K cooler than over the prairie while the maximum cooling was 2 K at 15 m and unmeasurable at 75 m. Our own large-eddy simulations of a convective boundary layer over a surface with sinusoidal perturbations in the heat flux (Cotton et al. 1987) have surface temperature perturbations of amplitude 4 K and wavelength 1500 m, but the temperature perturbation at the lowest grid level ($z = 30$ m) is typically only 0.2 K. (The temperature difference between this level and the surface is estimated from the prescribed heat flux using the Businger et al. 1971 formulation of surface layer profiles.)

At risk of over-generalizing we summarize the results discussed in the previous paragraph as follows: surface temperature variations on scales of a few kilometers can be of order 10 K, but the variations produced in air temperature above a few tens of meters are an order of magnitude less. We therefore believe it is possible—and maybe conceptually useful—to define a $\theta^{(2)}$ field with maximum magnitude (of order 10 K) at the surface and with z_S not more than a few tens of meters, such that the residual $\theta^{(1)}$ field has magnitude of order 1 K or less. In other words we define the $\theta^{(2)}$ field to capture the surface temperature fluctuations and the strong temperature gradients immediately above, while the $\theta^{(1)}$ field captures the smaller fluctuations in the rest of the boundary layer (and maybe in the overlying stable layer as well). Like Moeng and Wyngaard we will not attempt to speculate on the form of the $\theta^{(1)}$ or $p_B^{(1)}$ fields. Assuming that the $\theta^{(2)}$ field has

$$\int_0^{z_S} \Theta dz \sim (5 \text{ K}) \times (10 \text{ m}), \tag{15}$$

the fluctuations in the $p_B^{(2)}$ field at the surface will have typical magnitude $\sim 10^0$ Pa. On this basis there is no reason to believe a priori that surface temperature fluctuations in a convective boundary layer will have a major direct effect on the pressure field. The indirect effects, via changes in velocity and temperature throughout the depth of the boundary layer, are another matter.

Finally, we would like to point out a minor error in Moeng and Wyngaard's paper: their Figs. 2 and 3 are mislabeled. We believe that Fig. 2 shows contributions to the budget of the flux in "red dye" (not potential temperature) and that Fig. 3 applies to potential temperature (not red dye).

Acknowledgments. This work was supported by the Electric Power Research Institute under Contract 1630-53, by the U.S. Army Research Office under Contract DAAL03-86-K-0175 and by the New Zealand Meteorological Service.

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