An Analytical Study of the Sea Breeze

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ABSTRACT

In this paper we present theoretical results concerning the sea breeze intensity and its inland penetration as a function of latitude and friction. We produce solutions for the spatial structure of the streamfunction and momentum components and for their time behavior in the case of a step function forcing of finite duration, and in the case of periodic forcing. Results show that the sea breeze initially has an aspect ratio equal to unity and that earth rotation and friction affect its intensity but they are unimportant in determining its early geometry. The sea breeze has a characteristic time scale which is a combination of the inertial period and the *e*-folding time due to friction. For a time larger than this characteristic time scale, the inland penetration of the sea breeze is confined by a Rossby deformation radius, which includes a frictional effect. In fact, friction and inertia reduce, not only the intensity, but also the horizontal scale of motion; at the equator the controlling parameter for the intensity and penetration of the sea breeze is friction. When the friction is small, periodicity in the forcing enhances the intensity and the horizontal scale; however when the friction *e*-folding time is of the order of one day, the opposite is true. The existence of wave structure below 30 degrees of latitude, suggested by Rotunno, strongly depends on relative values of periodicity and friction, i.e., periodicity tends to make the governing partial differential equation for the streamfunction hyperbolic while friction tends to keep it elliptical; from the competition of these two contrasting effects waves may or may not result.

1. Introduction

The sea–land breeze, a common phenomenon in coastal areas at all latitudes, has been theoretically studied extensively (Defant 1951; Hawritz 1959; Mak and Walsh 1976) and its physics is quite well understood. Starting from previous analytical studies, we revisit the linear theory in rather general terms in order to examine the contributions of the effect of earth rotation and friction on the onset, the transient, the asymptotic and the periodic behavior of the sea breeze. In this sense the present paper is an extension of Rotunno (1983). He presented the sea breeze linear theory under periodic forcing; we have added an analysis for the nonperiodic forcing and we further comment on the periodic behavior in a dissipative system. Our results differ from Rotunno's for latitudes below 30 degrees, in that the inclusion of friction lowers the latitude at which the system switches from elliptical to hyperbolic, i.e., from a closed sea breeze vortex to propagating waves. For realistic values of friction, only an elliptic solution results. Observations of sea breezes at low latitudes in the peninsula of Carpenteria (Smith et al. 1981) support this last hypothesis. An extensive investigation by Yan and Anthes (1987), performed with a sophisticated nonlinear model, also confirms these results. In our paper we show that, even in a linear model, the influence of turbulent friction eliminates indefinite inland penetration, unless the winds are decoupled from the surface (i.e., such as can occur at night under radiational cooling, or after rainfall).

The theory and the mathematical approach, presented here, goes beyond the sea–land breeze application; for instance, it can be used to study the enhancement of convective precipitation by mesoscale variations in vegetative covering (Anthes 1984) or to study the modification of the mesoscale atmospheric flow due to irregular irrigation patterns.

2. The analytical approach

From Rotunno (1983) the linearized equations describing the sea–land breeze flow are

$$\left(\frac{\partial}{\partial t} + \lambda\right)u - fv + \frac{\partial}{\partial x}\Phi = 0 \qquad (2.1)$$

$$\left(\frac{\partial}{\partial t} + \lambda\right)v + fu = 0 \qquad (2.2)$$

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$$\left(\frac{\partial}{\partial t}+\lambda\right)w-b+\frac{\partial}{\partial z}\Phi=0 \quad (2.3)$$

$$\left(\frac{\partial}{\partial t}+\lambda\right)b+N^2w=Q \quad (2.4)$$

$$\frac{\partial}{\partial x}u+\frac{\partial}{\partial z}w=0, \quad (2.5)$$

where f is the Coriolis parameter, b is the buoyancy force, Φ is the geopotential, Q is the diabatic heat forcing which warms the air during the day and cools it during the night, λ^{-1} is the damping time due to friction, and N is the Brunt–Väisäla frequency. In this paper we use the following value for friction:

$$\lambda=1.2\omega \quad \omega=\frac{2\pi}{1\ \text{day}}. \quad (2.6)$$

The term ω denotes the diurnal pulsation.

Defining the streamfunction as

$$u=\frac{\partial}{\partial z}\psi \quad w=-\frac{\partial}{\partial x}\psi, \quad (2.7)$$

the primitive equations (2.1)–(2.5), using (2.7), can be reduced to a single equation for the streamfunction:

$$\left[\left(\frac{\partial}{\partial t}+\lambda\right)^2+N^2\right]\frac{\partial^2}{\partial x^2}\psi + \left[\left(\frac{\partial}{\partial t}+\lambda\right)^2+f^2\right]\frac{\partial^2}{\partial z^2}\psi=-\frac{\partial}{\partial x}Q. \quad (2.8)$$

a. The nondimensional equation for the streamfunction

We then define the following nondimensional quantities:

$$T=\frac{1}{\sqrt{f^2+\lambda^2}}; \quad \tau=tT^{-1}$$

$$\tilde{f}=fT; \quad \tilde{N}=NT; \quad \tilde{\lambda}=\lambda T; \quad s=\frac{\partial}{\partial\tau};$$

$$p=s+\tilde{\lambda}; \quad \tilde{\omega}=\omega T$$

$$\eta=z/h; \quad \xi=\left(\frac{\tilde{f}^2+p^2}{\tilde{N}^2+p^2}\right)^{1/2}\frac{x}{h}\approx\frac{\sqrt{\tilde{f}^2+p^2}}{\tilde{N}}\frac{x}{h},$$

where

$$\hat{f}(s)=L\{f(\tau)\}=\int_0^\infty f(\tau)\exp(-s\tau)d\tau \quad \text{and}$$

$$LL^{-1}\{f(\tau)\}=f(\tau)$$

$$\hat{\psi}=L\{\psi h^{-2}T\}; \qquad \tilde{\psi}=\psi h^{-2}T$$

$$(\hat{u},\hat{v},\hat{w})=L\{(u,v,w)h^{-1}T\};$$

$$(\tilde{u},\tilde{v},\tilde{w})=(u,v,w)h^{-1}T$$

$$\hat{b}=L\{bh^{-1}T^2\}; \qquad \tilde{b}=bh^{-1}T^2$$

$$\hat{Q}=L\{Qh^{-1}T^3\}; \qquad \tilde{Q}=Qh^{-1}T^3$$

$$\hat{\Phi}=L\{\Phi h^{-2}T^2\}; \qquad \tilde{\Phi}=\Phi h^{-2}T^2$$

$$\beta(p)=\left(\frac{1}{\tilde{N}^2+p^2}\right)^{1/2}\left(\frac{1}{\tilde{f}^2+p^2}\right)^{1/2}\approx\frac{1}{\tilde{N}}\frac{1}{\sqrt{\tilde{f}^2+p^2}}. \quad (2.9)$$

The characteristic time scale of the sea breeze T decreases with increasing latitude and friction. Friction keeps the time scale T finite at the equator, while h is the vertical scale of motion, i.e., the depth through which the diabatic heat Q is acting. The approximation in the horizontal coordinate ξ and in β is the hydrostatic approximation. The variables with the tilde are nondimensional; the variable with the hat are their Laplace transform. Using the definition stated in (2.9) for the nondimensional variables and for their Laplace transform, the streamfunction equation (2.8) can be written as

$$\frac{\partial^2}{\partial\xi^2}\hat{\psi}+\frac{\partial^2}{\partial\eta^2}\hat{\psi}=-\beta(p)\frac{\partial}{\partial\xi}\hat{Q}. \quad (2.10)$$

b. The Green functions

The Green function for the Poisson equation (2.10) in the upper semiplane, i.e., which satisfy the boundary condition $\hat{w}(\xi,\eta=0)=0$, is

$$g_\psi=-\frac{1}{2\pi}\ln\left[\frac{(\xi-\xi')^2+(\eta+\eta')^2}{(\xi-\xi')^2+(\eta-\eta')^2}\right]^{1/2}, \quad (2.11)$$

from which we deduce, through derivation, the Green functions for the horizontal and the vertical velocities:

$$g_u=\frac{\partial}{\partial\eta}g_\psi=\frac{1}{2\pi}\left[\frac{(\eta-\eta')}{(\xi-\xi')^2+(\eta-\eta')^2}-\frac{(\eta+\eta')}{(\xi-\xi')^2+(\eta+\eta')^2}\right] \quad (2.12)$$

$$g_w=-\frac{\partial}{\partial\xi}g_\psi=\frac{1}{2\pi}\left[\frac{(\xi-\xi')}{(\xi-\xi')^2+(\eta+\eta')^2}-\frac{(\xi-\xi')}{(\xi-\xi')^2+(\eta-\eta')^2}\right]. \quad (2.13)$$

3. The diabatic forcing and the transfer functions

a. The diabatic forcing distribution

To extend our analysis, let us now assume that the diabatic heat is distributed through a convective depth h over the horizontal region $0<x<\infty$. In our nondimensional units the forcing function is

$$\tilde{Q}=\tilde{Q}_0\,\text{He}(\xi)\,\text{He}(1-\eta)\tilde{q}(\tau)$$

$$\hat{Q}=\tilde{Q}_0\,\text{He}(\xi)\,\text{He}(1-\eta)\hat{q}(s), \quad (3.1)$$

where He is the Heaviside function.

b. The transfer functions

If $\tilde{q}(\tau) = \delta(\tau)$ and $\hat{q}(s) = 1$, the sea breeze response is

$$\hat{G}_\psi(\xi, \eta, p) = \frac{\beta(p)}{2\pi}\Big\{(\eta + 1)\ln(\xi^2 + (\eta + 1)^2)^{1/2} - 2\eta\ln(\xi^2 + \eta^2)^{1/2} + (\eta - 1) \times \ln(\xi^2 + (\eta - 1)^2)^{1/2} + \xi\Big[\tan^{-1}\Big(\frac{\eta + 1}{\xi}\Big) - 2\tan^{-1}\Big(\frac{\eta}{\xi}\Big) + \tan^{-1}\Big(\frac{\eta - 1}{\xi}\Big)\Big]\Big\}$$

$$\hat{G}_u(\xi, \eta, p) = \frac{\beta(p)}{2\pi}\{\ln(\xi^2 + (\eta + 1)^2)^{1/2} - 2\ln(\xi^2 + \eta^2)^{1/2} + \ln(\xi^2 + (\eta - 1)^2)^{1/2}\}$$

$$\hat{G}_w(\xi, \eta, p) = -\frac{\beta(p)}{2\pi}\Big[\tan^{-1}\Big(\frac{\eta + 1}{\xi}\Big) - 2\tan^{-1}\Big(\frac{\eta}{\xi}\Big) + \tan^{-1}\Big(\frac{\eta - 1}{\xi}\Big)\Big]. \quad (3.2)$$

The spatial structure of the transfer functions $\hat{G}$, normalized in amplitude $\beta = 1$ and in the nondimensional coordinate system, is illustrated in Fig. 1. We see that the sea breeze confined in a region of about two vertical units and two horizontal units. The Rossby deformation radius in these nondimensional coordinates is

$$\hat{R} = \Big(\frac{\tilde{N}^2 + p^2}{\tilde{f}^2 + p^2}\Big)^{1/2} \approx \frac{\tilde{N}}{\sqrt{\tilde{f}^2 + p^2}}. \quad (3.3)$$

In physical dimensional space it means that onshore flow is vertically confined within h and that the return flow is vertically confined within $2h$; the inland penetration is of the order of one Rossby deformation radius R.

Given the transfer functions $\hat{G}$, we can compute the sea breeze response for any time dependence of the diabatic forcing:

$$\hat{\psi} = \hat{G}_\psi\tilde{Q}_0\hat{q}(s); \quad \tilde{\psi} = \tilde{Q}_0\{\tilde{q}(\tau)*\tilde{G}_\psi(\tau)\}$$
$$\hat{u} = \hat{G}_u\tilde{Q}_0\hat{q}(s); \quad \tilde{u} = \tilde{Q}_0\{\tilde{q}(\tau)*\tilde{G}_u(\tau)\}$$
$$\hat{w} = \hat{G}_w\tilde{Q}_0\hat{q}(s); \quad \tilde{w} = \tilde{Q}_0\{\tilde{q}(\tau)*\tilde{G}_w(\tau)\}, \quad (3.4)$$

where $\{\tilde{q}(\tau)*\tilde{G}(\tau)\}$ denotes a product in the Faltung theorem sense (Fodor 1965):

$$\{\tilde{q}(\tau)*\tilde{G}(\tau)\} = \int_0^\tau \tilde{G}(\tau - u)\tilde{q}(u)du.$$

The time dependent transfer functions in (3.4) are

$$\tilde{G}_\psi(\xi, \eta, \tau) = \frac{\exp(-\tilde{\lambda}\tau)}{2\pi\tilde{N}}\int_0^\tau du\frac{1}{u}J_0(\tilde{f}\sqrt{\tau^2 - u^2})\tilde{F}_\psi(\xi, \eta, u)$$

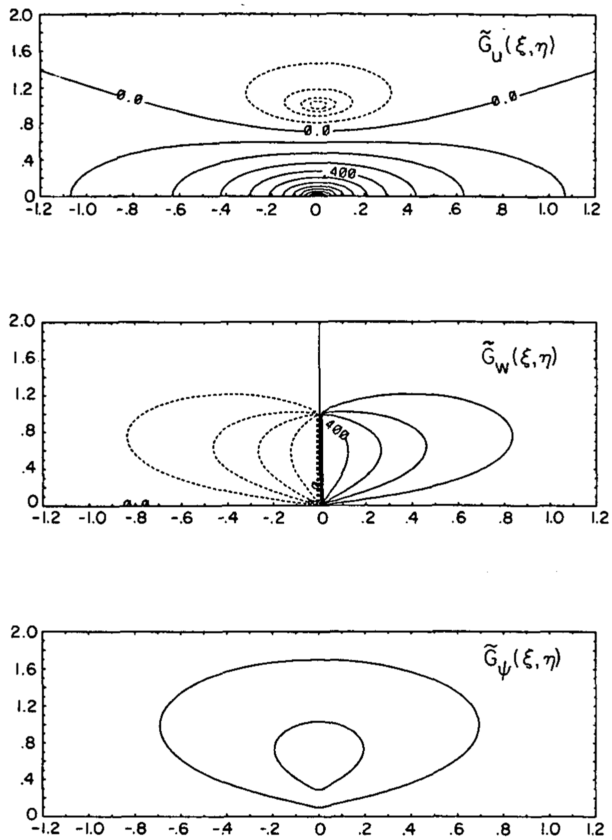

FIG. 1. The streamfunction ψ, the horizontal momentum component u, and the vertical momentum component w in nondimensional coordinates and normalized amplitude. The contours intervals are in units of 0.1.

$$\tilde{G}_u(\xi, \eta, \tau) = \frac{\exp(-\tilde{\lambda}\tau)}{2\pi\tilde{N}}\int_0^\tau du\frac{1}{u}J_0(\tilde{f}\sqrt{\tau^2 - u^2})\tilde{F}_u(\xi, \eta, u)$$

$$\tilde{G}_w(\xi, \eta, \tau) = -\frac{\exp(-\tilde{\lambda}\tau)}{2\pi\tilde{N}}\int_0^\tau du\frac{1}{u}J_0(\tilde{f}\sqrt{\tau^2 - u^2})\tilde{F}_w(\xi, \eta, u) \quad (3.5)$$

with

$$\tilde{F}_\psi = 2\eta\cos u_0 - (\eta + 1)\cos u_{0+} - (\eta - 1)\cos u_{0-} + \frac{\xi}{u}[u_{0+}\cos u_{0+} - \sin u_{0+} - 2(u_0\cos u_0 - \sin u_0) + u_{0-}\cos u_{0-} - \sin u_{0-}]$$

$$\tilde{F}_u = 2\cos u_0 - \cos u_{0+} - \cos u_{0-}$$
$$\tilde{F}_w = \sin u_{0+} - 2\sin u_0 + \sin u_{0-}.$$

In order to keep the time-dependent transfer functions reasonably simple, and because the sea breeze is nonhydrostatic only in its very early stage, we made the hydrostatic approximation in (3.5):

$$\eta = \frac{z}{h} \quad \xi = \frac{1}{TN}\frac{x}{h}; \quad u_0 = \frac{\eta}{\xi}u$$

$$u_{0+} = \frac{\eta+1}{\xi}u; \quad u_{0-} = \frac{\eta-1}{\xi}u. \quad (3.6)$$

In (3.5) and (3.6) u is an integration variable. The importance of the transfer functions is that they determine how the spatial structure of the sea breeze evolves in time. In (3.5), the sea breeze goes from its initial state of rest to its asymptotic state through a series of damped inertial-gravity waves, (gravity waves at the equator), expressed by the combination of the zero order Bessel function and the sines and the cosines.

4. The onset, the transient and the asymptotic behavior of the sea breeze under aperiodic forcing

In this section we examine the sea breeze response when the time dependence of the forcing in Eq. (3.1), $\tilde{q}(\tau)$, is a step function:

$$\tilde{q}(\tau) = \mathrm{He}(\tau) - \mathrm{He}(\tau - \tilde{a})$$

$$\hat{q}(s) = \frac{1}{s}[1 - \exp(-\tilde{a}s)]. \quad (4.1)$$

In (4.1) the diabatic forcing is zero before $t = 0$, it is constant when $0 < t < a$ and is again zero after $t = a$; which is a crude parameterization of the warming occurring over the land during a daylight of a duration.

a. The onset of the sea breeze $t \ll T$

First we examine the sea breeze behavior when time t is small in comparison to the sea breeze time scale T. We expand the streamfunction in a Taylor series near the time origin:

$$\tilde{\psi}(\xi, \eta, \tau) = \tilde{\psi}(\xi, \eta, \tau = 0) + \tau\frac{\partial}{\partial\tau}\tilde{\psi}(\xi, \eta, \tau = 0) + \frac{\tau^2}{2}\frac{\partial^2}{\partial\tau^2}\tilde{\psi}(\xi, \eta, \tau = 0) + \cdots, \quad (4.2)$$

where

$$\tilde{\psi}(\xi, \eta, \tau = 0) = \lim_{s\to\infty} s\hat{\psi}$$

$$\frac{\partial}{\partial\tau}\tilde{\psi}(\xi, \eta, \tau = 0) = \lim_{s\to\infty} s^2\hat{\psi}$$

$$\frac{\partial^2}{\partial\tau^2}\tilde{\psi}(\xi, \eta, \tau = 0) = \lim_{s\to\infty} s^3\hat{\psi}.$$

From Eq. (4.2) we compute the nondimensional streamfunction for the forcing specified in Eqs. (3.1) and (4.1):

$$\tilde{\psi}_{\mathrm{He}} = \tilde{Q}_0\tilde{G}_\psi(\xi, \eta)\frac{\tau^2}{2}. \quad (4.3)$$

The sea breeze intensity is initially zero and increases quadratically in time. In dimensional units we have

$$\psi_{\mathrm{He}} = hQ_0\tilde{G}_\psi\frac{t^2}{2}; \quad u_{\mathrm{He}} = Q_0\tilde{G}_u\frac{t^2}{2}$$

$$w_{\mathrm{He}} = Q_0\tilde{G}_w\frac{t^2}{2}; \quad v_{\mathrm{He}} = -fQ_0\tilde{G}_u\frac{t^3}{3!}. \quad (4.4)$$

The transfer functions $\tilde{G}_\psi$, $\tilde{G}_u$ and $\tilde{G}_w$ in Eqs. (4.3) and (4.4) are those defined in Eq. (3.2), with

$$\beta = 1; \quad \eta = \frac{z}{h}; \quad \xi = \frac{x}{h}. \quad (4.5)$$

Consequently the transfer functions in the early stage do not depend on time, latitude or friction. Only the along-shore component of the wind is latitude dependent. From Eq. (4.5) we see that the sea breeze is initially nonhydrostatic with an aspect ratio A equal to unity and a Rossby deformation radius R equal to the vertical scale of the diabatic forcing:

$$A = 1; \quad R = h. \quad (4.6)$$

The spatial structure of the sea breeze in nondimensional coordinate is shown in Fig. 1.

From Eqs. (4.3)–(4.5) and Fig. 1 we see that the forerunners, described by Geisler and Bretherton (1969) in terms of gravity waves, must interfere in such a way to confine the sea breeze near the thermal discontinuity.

b. The transient behavior of the sea breeze $t = O(T)$

In this section we examine how the sea breeze transits from its onset to its asymptotic state, i.e., when t is of the order of characteristic time T.

The sea breeze time behavior is given by the Faltung product between the forcing (4.1) and the transfer functions given in (3.5), as defined in (3.4).

Figure 2 shows the transient behavior of the sea breeze under a step function forcing of $a = 0.8$ day at 40° of latitude and at the equator. Results show that there is an increasing time lag between the forcing and the sea breeze as we move further inland. At the end of the daylight the sea breeze has reached an almost stationary state, $t \approx a$. After sunset, when the diabatic forcing is removed, the sea breeze rapidly decays with an e-folding time equal to λ^{-1}. The intensity of the sea breeze is almost zero when $t \approx a + 3\lambda^{-1}$. The difference between the onshore flow in the midlatitude regions and the onshore flow at the equator (there is no along-shore flow at the equator) is that the intensity grows more rapidly at low latitudes and reaches higher values, and the sea breeze penetrates deeper inland.

c. The asymptotic behavior of the sea breeze $t \gg T$

When the time t is much larger than the characteristic time T the sea breeze reaches its asymptotic state,

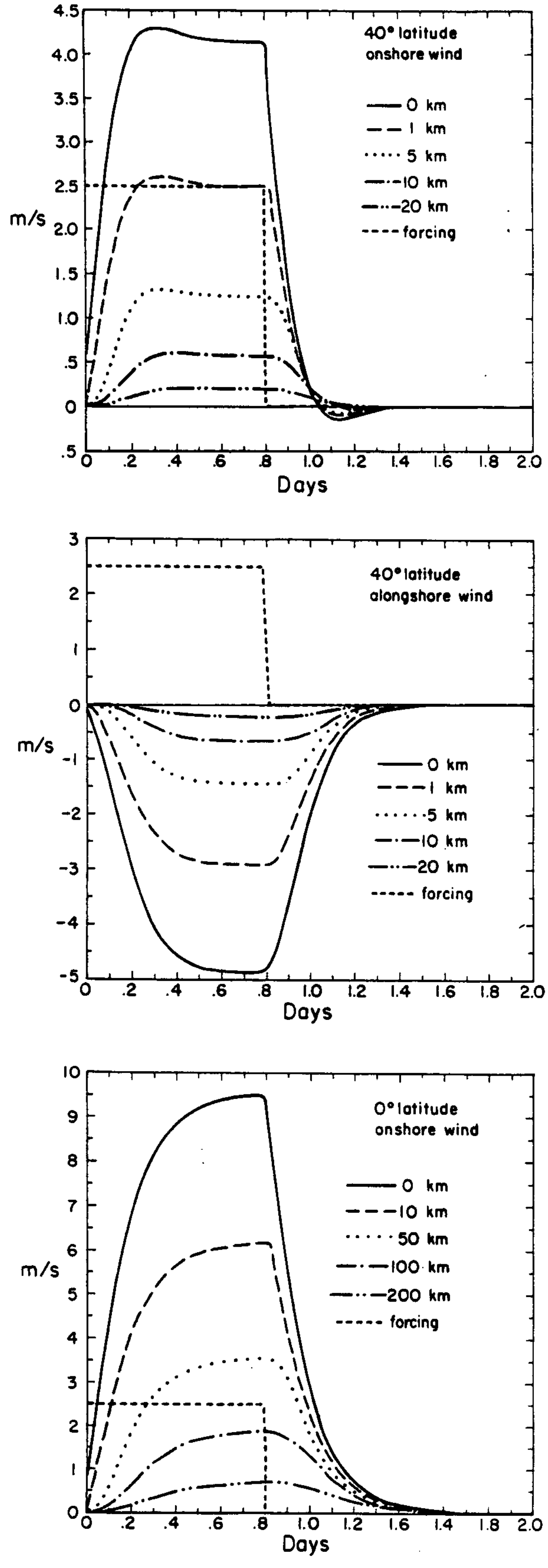

FIG. 2. The onshore wind intensity u and the alongshore wind intensity v for a step function forcing of $a = 0.8$ day duration ($\lambda = 1.2\omega$). At 40° latitude at inland distance $x = (0.0, 1.0, 5.0, 10.0, 20.0$ km); and at 0° latitude at inland distance $x = (0.0, 10.0, 50.0, 100.0, 200.0$ km).

which can be computed from Eq. (3.4) through the following limit:

$$\tilde{\psi}(\xi, \eta, \tau) = \lim_{s\to 0} s\hat{\psi}. \qquad (4.7)$$

We are interested in the well-developed state of the sea breeze, $t \approx a$. Since a is sufficiently larger than the characteristic time T because of friction, we can consider $t = a \approx \infty$, then we can make use of the limit (4.7) in Eq. (3.4):

$$\tilde{\psi}_{\rm He} = \tilde{Q}_0 \tilde{G}_\psi(\xi, \eta). \qquad (4.8)$$

The normalized nondimensional streamfunction and momentum components are shown in Fig. 1 in nondimensional coordinates. In dimensional units we have

$$\psi_{\rm He} = \psi_0 \tilde{G}_\psi; \quad u_{\rm He} = u_0 \tilde{G}_u;$$
$$w_{\rm He} = w_0 \tilde{G}_w; \quad v_{\rm He} = v_0 \tilde{G}_u; \qquad (4.9)$$

with

$$u_0 = T^2 Q_0; \quad \psi_0 = h u_0; \quad w_0 = \frac{h}{R_0} u_0; \quad v_0 = -\frac{f}{\lambda} u_0. \qquad (4.10)$$

The dimensional amplitudes are functions of latitude and friction; inertia and dissipation both contribute to reduce the sea breeze intensity through the characteristic time T. In Eqs. (4.8) and (4.9), the transfer functions are those defined in (3.2) with

$$\beta_0 = \frac{1}{TN}; \quad \eta = \frac{z}{h}; \quad \xi = \frac{x}{R_0} \qquad (4.11)$$

i.e., the amplitude of the asymptotic transfer functions is inversely proportional to the characteristic time and to the Brunt–Väisälä frequency. The actual intensity of the onshore wind is the product between u_0 and β_0, consequently the sea breeze is more intense for a weakly stratified atmosphere and, through the characteristic time, is less intense for increasing value of inertia and friction. Friction is more effective than inertia in controlling the intensity of the onshore flow and the inland penetration in low latitude regions. Furthermore the spatial structure of the transfer functions depends on inertia and friction. From (4.11) we see that the sea breeze has an aspect ratio A_0 and a Rossby deformation radius R_0:

$$A_0 = \left(\frac{f^2 + \lambda^2}{N^2 + \lambda^2}\right)^{1/2} \approx \frac{1}{TN} \quad R_0 = \frac{h}{A_0} \approx hTN, \qquad (4.12)$$

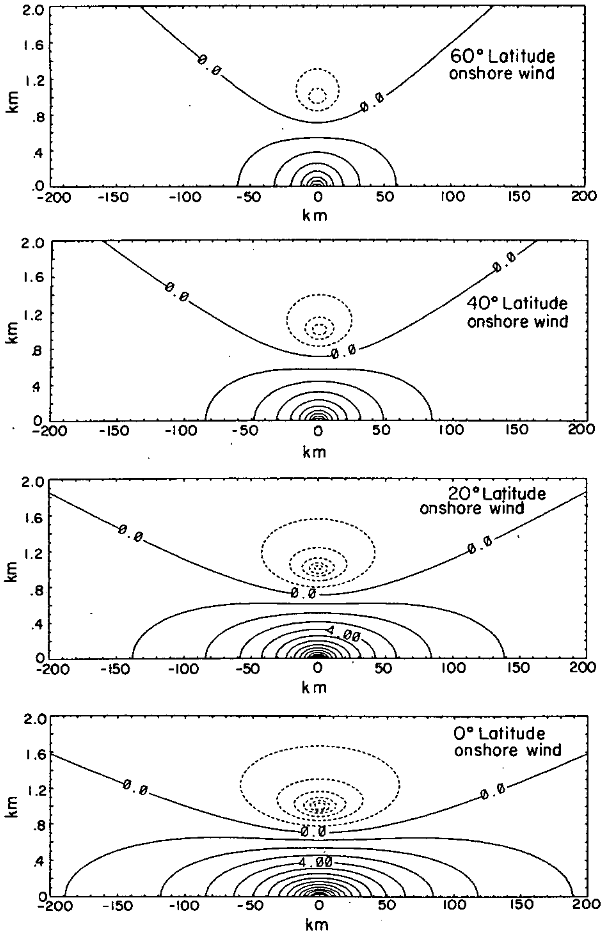

FIG. 3. The onshore wind u isolines at 0°, 20°, 40°, and 60° of latitude ($\lambda = 1.2\omega$); $\Delta u = 1$ m s^{-1}; horizontal and vertical distances are in kilometers. Asymptotic structure under step function forcing.

and the sea breeze penetration stops when

$$x > R_0, \tag{4.13}$$

i.e., when x is larger than the Rossby deformation radius corrected by the friction effect, which acts towards a reduction of the sea breeze inland penetration. The dimensional onshore momentum isolines for different latitudes are shown in Fig. 3; we see that the intensity of the onshore momentum decreases as the latitude increases. Figure 4 shows the corresponding alongshore wind behavior; the intensity of the alongshore momentum increases as the latitude increases. These behaviors are as expected, because the Coriolis force is more effective in feeding momentum into the alongshore component at the expense of the onshore component as the latitude increases. The horizontal extension of the onshore momentum and the alongshore momentum are comparable, because they are both bounded by the same Rossby deformation radius. Figure 5 shows the streamfunction under nonperiodic forcing at 0°, 20°, 40°, and 60° of latitude.

5. The onset, the transient and the asymptotic behavior of the sea breeze under periodic forcing

In this section we examine the sea breeze response when the time dependence of the diabatic forcing in Eq. (3.1) is periodic.

a. The onset of the sea breeze $t \ll T$

Since $T < \omega^{-1}$, the time behavior of a sinusoidal forcing can be approximate by its first-order expansion:

$$\tilde{q}(\tau) = \sin(\tilde{\omega}\tau) \approx \tilde{\omega}\tau; \quad \hat{q}(s) \approx \frac{\tilde{\omega}}{s^2}. \tag{5.1}$$

Then, from Eq. (4.2), we compute the nondimensional streamfunction for the forcing specified in Eqs. (3.1) and (5.1):

$$\tilde{\psi}_{\tilde{\omega}\tau} = \tilde{Q}_0 \tilde{G}_\psi(\xi, \eta) \frac{\tilde{\omega}}{3!} \tau^3. \tag{5.2}$$

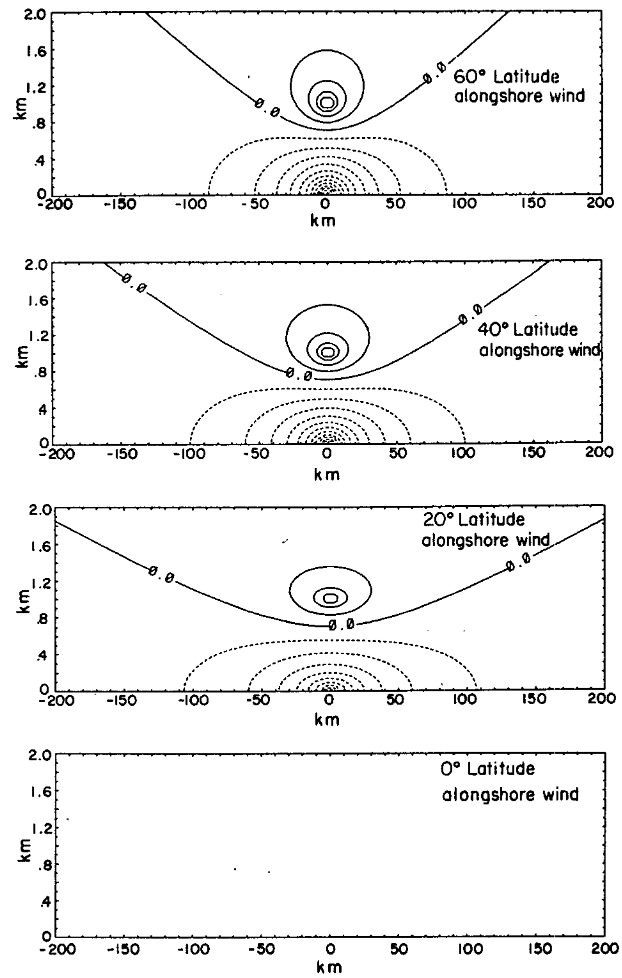

FIG. 4. The alongshore wind v isolines at 0°, 20°, 40°, 60° of latitude ($\lambda = 1.2\omega$); $\Delta v = 1$ m s^{-1}; horizontal and vertical distances are in kilometers. Asymptotic structure under step function forcing.

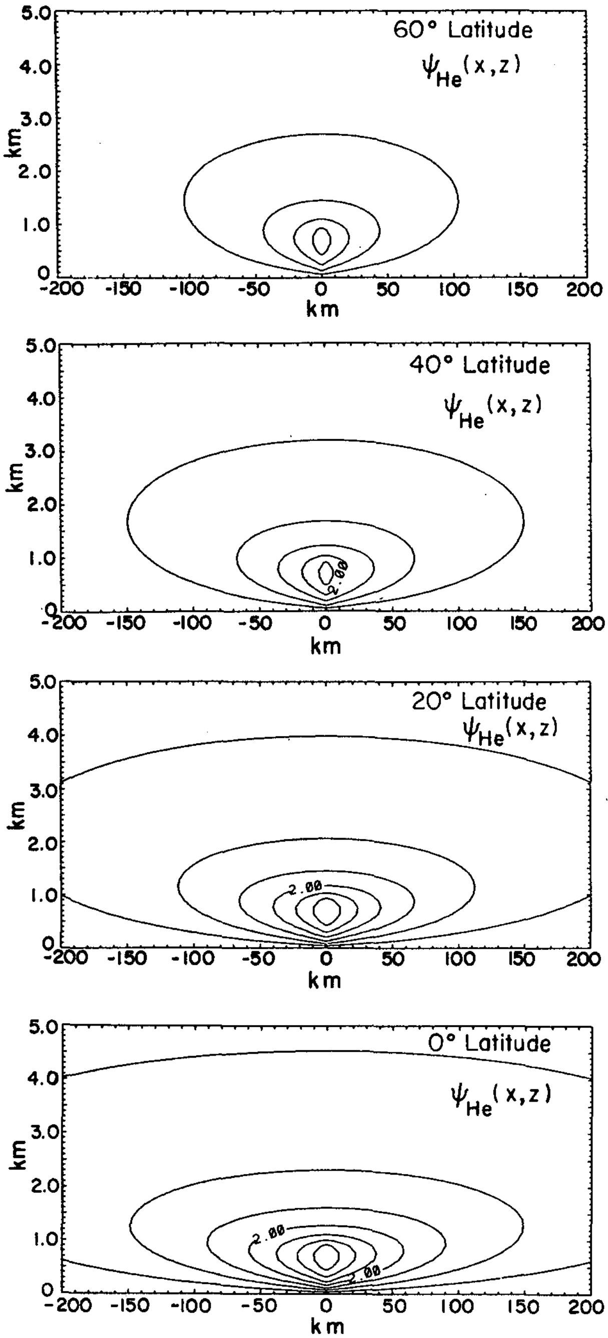

FIG. 5. The streamfunction ψ asymptotic isolines, under step function forcing and at 0°, 20°, 40°, 60° of latitude ($\lambda = 1.2\omega$); $\Delta\psi = 500\ \mathrm{m^2\ s^{-1}}$; horizontal and vertical distances are in kilometers.

The sea breeze intensity increases with the third power of time. In dimensional units we have

$$\psi_{\tilde{\omega}\tau} = \frac{1}{T} hQ_0\tilde{G}_\psi \frac{\tilde{\omega}}{3!} t^3; \quad u_{\tilde{\omega}\tau} = \frac{1}{T} Q_0\tilde{G}_u \frac{\tilde{\omega}}{3!} t^3;$$

$$w_{\tilde{\omega}\tau} = \frac{1}{T} Q_0\tilde{G}_w \frac{\tilde{\omega}}{3!} t^3; \quad v_{\tilde{\omega}\tau} = -f\frac{1}{T} Q_0\tilde{G}_u \frac{t^4}{4!}. \quad (5.3)$$

The transfer functions $\tilde{G}_\psi$, $\tilde{G}_u$ and $\tilde{G}_w$ in Eqs. (5.2) and (5.3) are those defined in Eq. (3.2), with

$$\beta = 1; \quad \eta = \frac{z}{h}; \quad \xi = \frac{x}{h}; \quad (5.4)$$

i.e., the transfer functions in the early stage do not depend on time, latitude, friction or pulsation of the forcing. Only the alongshore component of the wind is latitude dependent. From Eq. (5.4) we see that the sea breeze is initially nonhydrostatic with an aspect ratio A equal to unity and a Rossby deformation radius R equal to the vertical scale of the diabatic forcing:

$$A = 1; \quad R = h. \quad (5.5)$$

b. The transient behavior of the sea breeze $t = O(T)$

The sea-breeze time behavior is given by the Faltung product, defined in (3.4), between the sinosoidal forcing

$$\tilde{q}(\tau) = \sin(\tilde{\omega}\tau); \quad \hat{q}(s) = \frac{\omega}{s^2 + \omega^2} \quad (5.6)$$

and the transfer functions given in (3.5).

Figure 6 shows the transient behavior of the sea breeze under sinusoidal forcing at 40° of latitude and at the equator. Results shows that the periodic response is reached after a time $t \approx 3T$. Because of the strong damping introduced by friction, this time can be shorter than a day. Comparing Fig. 2 with Fig. 6, we note that the intensity of the sea breeze in the periodic regime is smaller than in the aperiodic regime in the low and in the midlatitude regions, which contradicts the expected behavior according to Rotunno.

c. The periodic response $t \gg T$

The asymptotic periodic behavior, studied by Rotunno and (1983) and Niino (1987) is reached for time $t > 3\lambda^{-1} = O(3\omega^{-1})$ (Fig. 6). In the absence of friction, several days may be needed to have a periodic response because of the presence of undamped inertial waves. To examine the asymptotic periodic behavior of the sea breeze, we rewrite the forcing as

$$\tilde{q}(\tau) = \exp(i\tilde{\omega}\tau); \quad \hat{q}(s) = \frac{1}{s - i\tilde{\omega}}. \quad (5.7)$$

From Eq. (2.8), the governing equation is

$$[N^2 - \omega^2 + \lambda^2]\frac{\partial^2}{\partial x^2}\psi + [f^2 - \omega^2 + \lambda^2]\frac{\partial^2}{\partial z^2}\psi + 2i\omega\lambda\left\{\frac{\partial^2}{\partial x^2}\psi + \frac{\partial^2}{\partial z^2}\psi\right\} = -Q_0\delta(x)\,\mathrm{He}(h - z). \quad (5.8)$$

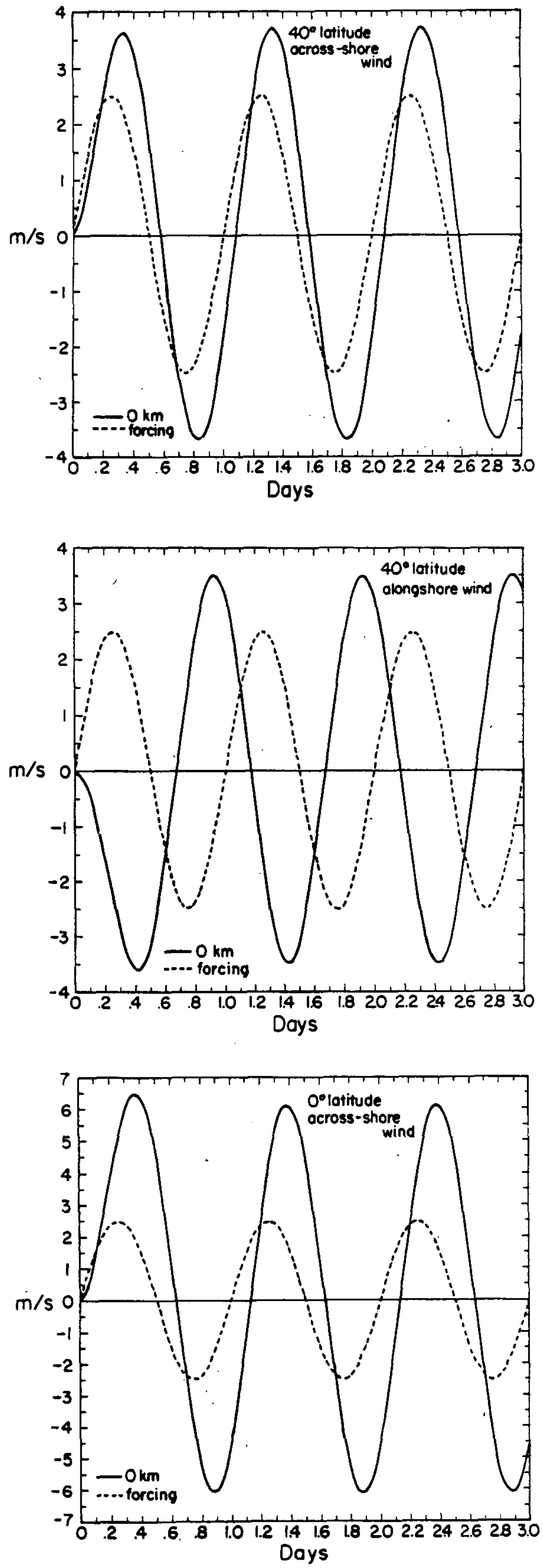

FIG. 6. The intensity of the across-shore wind u and the intensity of the alongshore wind v for a periodic in-time forcing ($\lambda = 1.2\omega$), at 40° latitude and at 0° latitude at the coastline.

1) THE SEA BREEZE BEHAVIOR WHEN $f^2 + \lambda^2 - \omega^2 > 0$

To solve Eq. (5.8) we pose:

$$\psi = \psi_r + i\psi_i. \tag{5.9}$$

Using (5.9) in Eq. (5.8) and equating the real and imaginary parts separately, we have the following two equations in nondimensional form:

$$\frac{\partial^2}{\partial \xi^2}\tilde{\psi}_r + \frac{\partial^2}{\partial \eta^2}\tilde{\psi}_r - \gamma\left\{\frac{\partial^2}{\partial \zeta^2}\tilde{\psi}_i + \frac{\partial^2}{\partial \eta^2}\tilde{\psi}_i\right\} = -\beta_\omega \tilde{Q}_0 \delta(\xi)\,\mathrm{He}(1-\eta) \tag{5.10}$$

$$\frac{\partial^2}{\partial \xi^2}\tilde{\psi}_i + \frac{\partial^2}{\partial \eta^2}\tilde{\psi}_i + \gamma\left\{\frac{\partial^2}{\partial \zeta^2}\tilde{\psi}_r + \frac{\partial^2}{\partial \eta^2}\tilde{\psi}_r\right\} = 0, \tag{5.11}$$

with

$$\beta_\omega = \frac{1}{\sqrt{\tilde{N}^2 + \tilde{\lambda}^2 - \tilde{\omega}^2}}\frac{1}{\sqrt{\tilde{f}^2 + \tilde{\lambda}^2 - \tilde{\omega}^2}} \approx \beta_0 \frac{T_\omega}{T}$$

$$\eta = \frac{z}{h}; \quad \xi = \alpha\frac{x}{h} = \alpha\zeta; \quad \zeta = \frac{x}{h};$$

$$T_\omega = \frac{1}{\sqrt{f^2 + \lambda^2 - \omega^2}}$$

$$\alpha = \left(\frac{\tilde{f}^2 + \tilde{\lambda}^2 - \tilde{\omega}^2}{\tilde{N}^2 + \tilde{\lambda}^2 - \tilde{\omega}^2}\right)^{1/2} \approx \beta_0 \frac{T}{T_\omega} = \mathrm{O}(10^{-2});$$

$$\gamma = 2T_\omega^2 \omega\lambda = \mathrm{O}(1). \tag{5.12}$$

Here β_0 is the amplitude of the asymptotic transfer function under aperiodic forcing. Equations (5.10) and (5.11), through some rearrangement, lead to the following two equations:

$$\frac{\partial^2}{\partial \xi'^2}\tilde{\psi}_r + \frac{\partial^2}{\partial \eta'^2}\tilde{\psi}_r = -\beta'_\omega \tilde{Q}_0 \delta(\xi')\,\mathrm{He}(1-\eta') \tag{5.13}$$

$$\tilde{\psi}_i(\xi', \eta') = -\gamma\tilde{\psi}_r\left(\frac{\xi'}{\alpha}, \eta'\right), \tag{5.14}$$

where (5.13) is for the streamfunction in-phase with the forcing and (5.14) is for the out-of-phase streamfunction, with

$$\eta' = \eta; \quad \xi' = \xi\left(\frac{1+\gamma^2}{1+\gamma^2\alpha^2}\right)^{1/2} \approx \xi\sqrt{1+\gamma^2}$$

$$= \frac{x}{R_0}\frac{T^2}{T_\omega T'_\omega}; \quad T'_\omega = \left[\frac{f^2 + \lambda^2 - \omega^2}{(f^2 + \lambda^2 - \omega^2)^2 + 4\omega^2\lambda^2}\right]^{1/2}$$

$$\beta'_\omega = \beta_\omega\left[\frac{1}{(1+\gamma^2)(1+\gamma^2\alpha^2)}\right]^{1/2}$$

$$\approx \beta_\omega \frac{1}{\sqrt{1+\gamma^2}} = \beta_0 \frac{T_\omega T'_\omega}{T^2}. \tag{5.15}$$

The out-of-phase streamfunction is proportional to the in-phase streamfunction through a factor γ. Furthermore the factor α in the horizontal coordinate, Eq. (5.14), confines the out-of-phase streamfunction (and the phase lag) to a region which is typically two order of magnitude smaller than the horizontal extension of the sea breeze. Equation (5.13) is formally equal to Eq. (2.10), which we have already extensively studied. In dimensional units we have

$$\psi_r = \psi_{0r}\tilde{G}_{\psi_r}(\xi', \eta'); \quad u_r = u_{0r}\tilde{G}_{u_r}(\xi', \eta'); \quad w_r = w_{0r}\tilde{G}_{w_r}(\xi', \eta')$$

$$\psi_i = \psi_{0i}\tilde{G}_{\psi_r}\left(\frac{\xi'}{\alpha}, \eta'\right); \quad u_i = u_{0i}\tilde{G}_{u_r}\left(\frac{\xi'}{\alpha}, \eta'\right); \quad w_i = w_{0i}\tilde{G}_{w_r}\left(\frac{\xi'}{\alpha}, \eta'\right) \quad (5.16)$$

$$u_{0r} = u_0 = T^2Q_0; \quad \psi_{0r} = hu_0; \quad w_{0r} = \frac{h}{R_r}u_0$$

$$u_{0i} = -\gamma u_0; \quad \psi_{0i} = -\gamma hu_0; \quad w_{0i} = -\gamma\frac{h}{R_i}u_0. \quad (5.17)$$

The phase lag φ is a function of space:

$$\varphi = \tan^{-1}\frac{\psi_i}{\psi_r}. \quad (5.18)$$

The transfer functions are those given (3.2), with the amplitude and the coordinates defined in (5.15). From Eqs. (5.16) and (5.15) we see that the sea breeze has an aspect ratio A_r for the in-phase and an aspect ratio A_i for the out-of-phase flow:

$$A_r = \left(\frac{1+\gamma^2}{1+\gamma^2\alpha^2}\right)^{1/2}\left(\frac{\tilde{f}^2+\tilde{\lambda}^2-\tilde{\omega}^2}{\tilde{N}^2+\tilde{\lambda}^2-\tilde{\omega}^2}\right)^{1/2}$$

$$\approx A_0\frac{T^2}{T_\omega T'_\omega} = O(10^{-2}); \quad A_i \approx \frac{T}{T'_\omega} = O(1) \quad (5.19)$$

where A_0 is the asymptotic aspect ratio under aperiodic forcing. The Rossby deformation radius R_r for the in-phase and R_i for the out-of-phase flow are

$$R_r = R_0\frac{T_\omega T'_\omega}{T^2}; \quad R_i = h\frac{T'_\omega}{T} \quad (5.20)$$

where R_0 is the asymptotic Rossby radius under aperiodic forcing. The in-phase flow vanishes when $|x| > R_r$ and the out-of-phase flow vanishes when $|x| > R_i$.

Since the actual amplitude of the in-phase across-shore wind is the product between u_{0r} and β'_ω and given the aspect ratio A_r, we see that periodicity increases the intensity and the horizontal extension of the sea breeze when

$$T^2 < T_\omega T'_\omega, \quad \text{i.e., when} \quad \lambda^2 < \frac{2f^2-\omega^2}{2}, \quad (5.21)$$

as stated by Rotunno for the nondissipative case.

When the dissipation is large, the periodicity

$$T^2 > T_\omega T'_\omega, \quad \text{i.e., when} \quad \lambda^2 > \frac{2f^2-\omega^2}{2}, \quad (5.22)$$

reduces the amplitude and the inland penetration of the sea breeze. The combined effect of periodicity and dissipation in reducing the amplitude can be seen comparing Fig. 2 with Fig. 5. Figure 7 shows the streamfunction in-phase with the periodic forcing at 0°, 20°, 40°, 60° of latitude. Figure 8 shows that the out-of-phase streamfunction at 0°, 20°, 40°, 60° of latitude; ψ_i has a limited horizontal extension at any latitude.

Like in the case of nonperiodic forcing, dissipation is more effective in reducing the horizontal extension of the sea breeze in the low latitudes regions.

2) Waves generated by periodic diabatic forcing when $\omega^2 - (f^2 + \lambda^2) > 0$ and $|x| > h$

When the dissipation is small ($\lambda^2 < \omega^2 - f^2$), waves can occur after few days of sea breeze below the following latitude:

$$0 \leqslant \text{latitude} = \sin^{-1}\frac{\sqrt{\omega^2-\lambda^2}}{2\omega} < 30°. \quad (5.23)$$

The presence of dissipation lowers the latitude at which waves occur; the nondissipative wave motion has been discussed by Rotunno (1983).

We briefly comment on these waves. Sufficiently far from the perturbing heating source, the contribution coming from the elliptic term in Eq. (5.8) can be neglected (see also Fig. 8) and the propagating wave part of the perturbation is described by the following hyperbolic equation (5.24):

$$\frac{\partial^2}{\partial\xi^2}\tilde{\psi} - \frac{\partial^2}{\partial\eta^2}\tilde{\psi} = -\beta_\omega\tilde{Q}_0\delta(\xi)\,\mathrm{He}(1-\eta)$$

$$\beta_\omega = \frac{1}{\tilde{N}}\frac{1}{\sqrt{\omega^2-(f^2+\lambda^2)}}$$

$$\eta = z/h \quad \xi = \frac{x}{h}\frac{\sqrt{\tilde{\omega}^2-(\tilde{f}^2+\tilde{\lambda}^2)}}{\tilde{N}}, \quad |x| > h. \quad (5.24)$$

In Eq. (5.24) we have made the hydrostatic approximation. The waves which satisfy to Eq. (5.24) under sinosoidal forcing are of the form

$$\tilde{\psi} = -\beta(\tilde{\omega})\tilde{Q}_0\sum_{n=1}^{\infty}\frac{4}{k_n^3}\sin(k_n\xi)\sin(k_n\eta)\sin(\tilde{\omega}\tau), \quad k_n = n\pi. \quad (5.25)$$

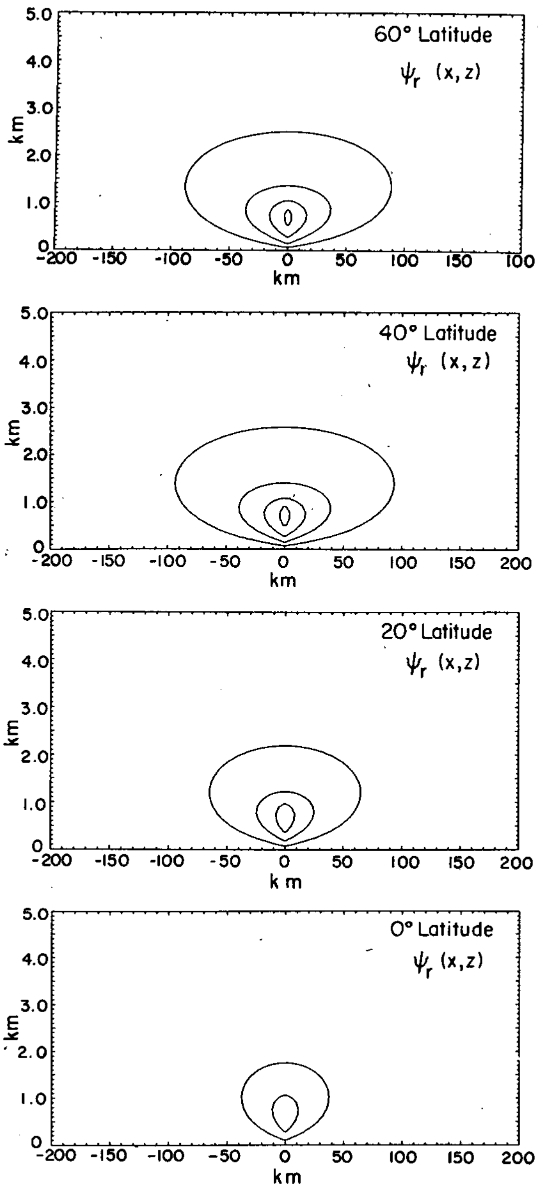

FIG. 7. The asymptotic isolines of the real part of the streamfunction ψ_r (in-phase with the forcing), under periodic forcing and at 0°, 20°, 40°, 60° of latitude ($\lambda = 1.2\omega$); $\Delta\psi_r = 500\ \mathrm{m^2\,s^{-1}}$; horizontal and vertical distances are in kilometers.

Sufficiently far from the source, in the space of non-dimensional coordinates, the waves propagate at $\pi/2$ from the diabatic forcing discontinuity with a phase lag $\varphi = \pi$ (Rotunno 1983).

6. Comments on the results

The choice of the characteristic time scale $T = (f^2 + \lambda^2)^{-1/2}$ is appropriate for the sea breeze because this relation not only greatly simplifies the formulas, but also shows clearly how inertia and friction both contribute to reduce the inland penetration and the intensity of the sea breeze. In the low latitude regions the major controlling factor for the amplitude and the horizontal extension of the flow is friction. The sea breeze is mainly a transient phenomenon and it should be studied accordingly. Stationary state or a pure periodic state is only reached after a time large in comparison to the e-folding time due to friction. In presence of periodic forcing, pulsation contributes, together with inertia and friction, to the Rossby radius and the amplitude of the flow. Periodicity in the forcing enhances

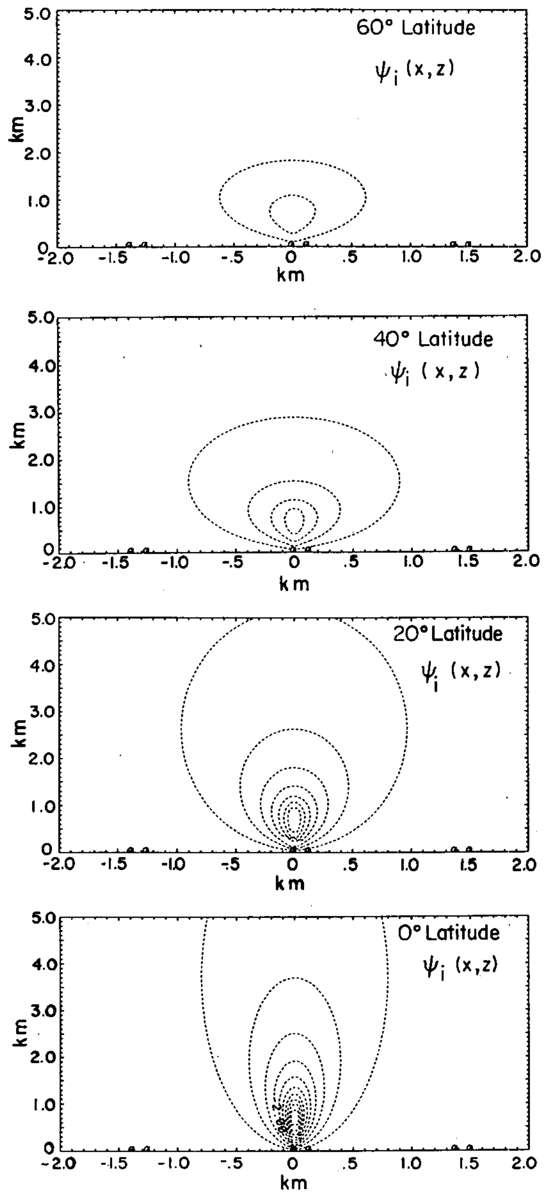

FIG. 8. The asymptotic isolines of the imaginary part of the streamfunction ψ_i (out-of-phase with the forcing), under periodic forcing and at 0°, 20°, 40°, 60° of latitude ($\lambda = 1.2\omega$); $\Delta\psi_i = 500\ \mathrm{m^2\,s^{-1}}$; horizontal and vertical distances are in kilometers.

the sea breeze intensity and the horizontal scale of motion only when the friction is smaller than the difference between the inertial and the forcing pulsation (Rotunno 1983). Otherwise the reverse is true. The wave pattern proposed by Rotunno below 30° of latitude under periodic forcing is likely to be a rare event because (i) except for situations where the thermally-forced wind is decoupled from the surface, turbulent friction will prevent the occurrence of propagating waves altogether and (ii) when friction is small or absent, a number of inertial periods are needed to reach the asymptotic state, and since the inertial periods are large at low latitudes, the sea breeze has to persist for several days in order to generate waves.

Finally the trapeze instability studied by Sun and Orlanski (1981), in connection with the convective instability observed at low latitude, will not occur when the friction e-folding time is shorter than 1 day$/2\pi$ (in this case the solution is limited and bounded at all latitudes). The observed cloud elements, organized into lines parallel to the coast several kilometers inland, in our opinion, are likely to be due to the large vertical velocity at the sea breeze front and to the instabilities which occur behind the head of the sea breeze gravity current.

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