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Multifractal model for eddy diffusivity and counter-gradient term in atmospheric turbulence

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Abstract

A new approach for eddy diffusivity and counter-gradient term in atmospheric turbulent fluxes is developed. This scheme is based on the Taylor statistical theory of turbulence and on a multifractal approach to the turbulent spectrum of energy. The non-extensive thermodynamics description is used to obtain a multifractal model. © 2001 Published by Elsevier Science B.V.

Keywords: Eddy diffusivity; Counter-gradient term; Taylor's theory; Multifractal approach; Non-extensive thermodynamics

1. Introduction

The atmosphere is not a homogeneous air mass around the Earth. It presents stratification. The first layer in the atmosphere is the planetary boundary layer (PBL), a thin layer in direct contact with the ground, where the turbulent and viscosity effects must be considered. The evolution of the PBL is controled by turbulent mixing induced by temperature difference between the atmosphere and Earth physical frontier (thermal production), and by the winds in the lower levels (mechanical production). In a convective boundary layer (CBL) the turbulence is generated by heat flux from

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the Earth to the atmosphere, and by wind field shear. The stable boundary layer (SBL) presents an inverse heat flux between the Earth and the atmosphere of that in the CBL. The SBL appears in the nocturnal period, it also occurs in some cloudy days or under other special conditions. In the neutral layer the turbulence is predominantly mechanical, that is, small heat flux and intensity of wind field relatively high.

Assuming Reynolds' hypothesis, where the turbulence is described as a sum of a mean stream plus a fluctuation term (with zero mean), the turbulence contribution in momentum, energy, and mass equations is constituted by the product between fluctuations. These terms represent new unknowns in the equations. The system of equations can be closed using the K-theory, where the turbulent fluxes are represented by the gradient of the mean stream multiplied by an eddy diffusivity. A counter-gradient term can also be added for a CBL. New formulations for these parameters in a CBL are derived based on the Taylor's statistical theory of turbulence [1] and on multifractal approach to the energy spectrum. Tsallis' description of the non-extensive thermodynamics [2] is used to derive a multifractal model for energy spectrum.

2. Modeling intermittency by multifractal approach

From objections to the Kolmogorov's classical theory, some alternative approaches have been proposed. One of them, the multifractal approach is applied [3]. In this scheme it is supposed that the eddies belonging to the energy cascade do not fill the space. Then, the structure function is supposed to be [3]

$$\frac{S_s(r)}{v_0^s} \equiv \frac{\langle v_r^s \rangle}{v_0^s} \sim \int_I \mathrm{d}\mu(h) \left(\frac{r}{L}\right)^{sh+3-D_F(h)}, \tag{1}$$

where the exponents lie in the interval $I = (h_{\min}, h_{\max})$, for each h in the inertial subrange; $v_r \equiv v(x+r) - v(x)$, $\langle\cdot\rangle$ is the mean value; L is the integral scale; and $D_F(h)$ is the fractal dimension. In this subrange: $r \ll L$, consequently the smallest exponent will dominate expression (1). Here, $\zeta_s = \inf\,\{sh + 3 - D_F(h)\}$ denotes the exponent for scaling the structure function.

Recently, the connection between intermittency and the non-extensive thermostatistics proposed by Tsallis [2] have been investigated [4–8]. This connection is due to the scaling properties of multifractal of this description of the thermodynamics. The theory developed by Ramos et al. (1999) [4] will be adopted. Therefore, the fractal dimension can be expressed as

$$D_F = 3\,\frac{\log\{[3 - q(r)]\,r/\eta\}}{\log(2r/\eta)}\,, \tag{2}$$

where $\eta \sim (v^3/\varepsilon)^{1/4}$ is the Kolmogorov's scale; and the parameter-q, which plays a central role in Tsallis' non-extensive description, is given by

$$q(r) = \frac{15 - 7H_L(r/L)^{\alpha}}{9 - 5H_L(r/L)^{\alpha}} = \frac{15 - 7H_\eta(r/\eta)^{\alpha}}{9 - 5H_\eta(r/\eta)^{\alpha}}\,, \tag{3}$$

with $\alpha = \zeta_4 - 2\zeta_2$, and $H_L = H_\eta\,(L/\eta)^\alpha = 3$. It is pointed out that for $q = 1$ the Boltzmann–Gibbs extensive statistics is recovered.

3. Eddy diffusivity and counter-gradient term

From Taylor statistical diffusion theory [1] an asymptotic expression, for large travel times ($t \to \infty$), for the eddy diffusivity in a PBL can be derived [9]

$$K_{\alpha\alpha} = \frac{\sigma_i^2\,\beta_i\,F_i(0)}{4}, \quad \text{with} \begin{cases} \alpha = x,\ y,\ z\ ; \\ i = u,\ v,\ w\ ; \end{cases} \tag{4}$$

where $\sigma_i^2 = \int_0^\infty S_i(n)\,\mathrm{d}n$ is the velocity variance; $\beta_i = (\sqrt{\pi}\,U)/(4\,\sigma_i)$ is the ratio between Langrangian and Eulerian time-scales; $F_i(n) = S_i(n)/\sigma_i^2$ is the normalized Eulerian spectrum of energy.

Following the procedure used in Ref. [10] and considering the expression for energy spectra from Eq. (1), a generalized expression for the normalized multifractal energy spectra can be derived:

$$F_i(n) = \frac{n\,S_i(n)}{\sigma_i^2} = \frac{f}{(f_m^*)_i}\left[1 + \frac{1}{\zeta_2}\frac{f}{(f_m^*)_i}\right]^{-(1+\zeta_2)}, \tag{5}$$

where $n = k\,U/2\pi$ is the relation between the frequency n and the wavelength k, U is the velocity of the main stream; $f = nz/U$ is the non-dimensional frequency, z is the height above the surface; and $(f_m^*)_i$ is the maximum value of the spectrum.

The turbulent fluxes in a CBL can be approximated using the eddy diffusivity and a counter-gradient term—firstly derived from the turbulent energy equation, considering laboratory measurements, by Deardoff [11]—as follows

$$\overline{v_i'\,\chi'} = -K_{\alpha\alpha}\left(\frac{\partial\chi}{\partial\alpha_i} - \gamma_\chi\right).$$

In atmospheric applications, the vertical component has a significant contribution. The expression presented by Cuijpers and Holtslag [12] is used here

$$\gamma_\chi = \frac{3}{2}\left(\frac{w_*}{\sigma_w}\right)^2\frac{\chi_*}{z_i}; \quad \text{with:}\ \ \chi_* = \frac{1}{w_*\,z_i}\int_0^{z_i}\overline{w'\,\chi'}\mathrm{d}z\,. \tag{6}$$

From Eqs. (4) and (5), and from expression for $(f_m^*)_i$ — see Ref. [10], follows the formula for vertical eddy diffusivity:

$$\frac{K_{zz}}{z_i\,w_*} = 0.15\,z_i^{\zeta_2/2-1/3}\,\Psi^{1/3}\left[1 - \mathrm{e}^{-4(z/z_i)} - 0.0003\,\mathrm{e}^{8(z/z_i)}\right]^{1+\zeta_2/2}, \tag{7}$$

where $\Psi = \varepsilon\,z_i/w_*^3$ is the nondimensional molecular dissipation rate function, with w_* the velocity scaling, and z_i the CBL height.

4. Final remarks

An extra term ($\sim z_i^{\zeta_2/2-1/3}$) was found for the eddy diffusivity derived from the multifractal approach. Considering $\zeta_2/2 - 1/3 \sim 1/40$ (as in [4]), and the CBL height

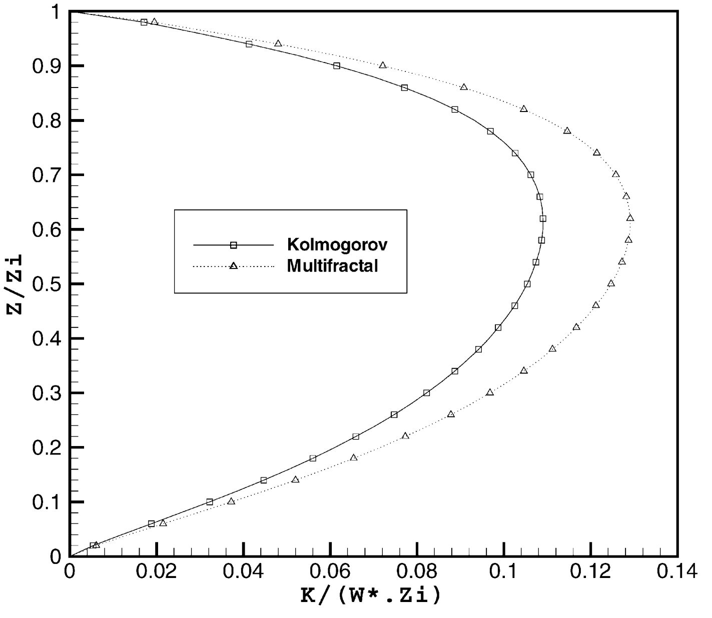

Fig. 1. Vertical eddy diffusivity for Kolmogorov's and multifractal approach.

of $O(10^3\,\mathrm{m})$, the eddy diffusivity from multifractal approach is 20% greater than the eddy diffusivity derived from classical Kolmogorov's hypothesis, see Fig. 1. Taking into account that $K_{\alpha\alpha}\sim\sigma_i$ and $\gamma\sim 1/\sigma_i^2$, then there will be a reduction in the value of the counter-gradient term for the multifractal approach.

The next step is to use the experimental data to evaluate whether our correction improves the meteorological fields predicted in numerical codes, such as those used in [13]. On the other hand, observational data can also be used for estimating the eddy diffusivities themselves [14].

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