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Representing intermittency in turbulent fluxes: An application to the stable atmospheric boundary layer

Haroldo F. Campos Velho[a,*], Reinaldo R. Rosa[a], Fernando M. Ramos[a], Roger A. Pielke Sr[b], Gervásio A. Degrazia[c]

[a]*Laboratório Associado de Computação e Matemática Aplicada (LAC), Instituto Nacional de Pesquisas Espaciais (INPE), Cx. Postal 515, 12201-970– São José dos Campos (SP), Brazil*
[b]*Department of Atmospheric Science, Colorado State University (CSU), Fort Collins, CO, USA*
[c]*Departamento de Física, Universidade Federal de Santa Maria (UFSM), Santa Maria, RS, Brazil*

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Abstract

A new formulation for eddy diffusivity is derived from Taylor's statistical theory on turbulence and from a generalized turbulent spectral equation for energy in the inertial subrange. The latter aspect is taken into account for considering the intermittency phenomenon within turbulence model. The approach is used for a stable atmospheric boundary layer parameterization.

Keywords: Eddy diffusivity; Intermittency; Taylor's theory; Atmospheric turbulence

1. Introduction

Intermittency concept has been proposed to explain some theoretical and experimental discrepancies from Kolmogorov's theory on turbulence [1]. Parisi

*Corresponding author. Fax: +55 12 3945 6375.
E-mail address: haroldo@lac.inpe.br (H.F. Campos Velho).

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and Frisch [2] used a multifractal approach to model intermittency, where the second Kolmogorov's hypothesis (self-similarity) is replaced by the assumption that "*turbulent flow is assumed to possess a range of scaling exponents* $b \in (b_{\min},\ b_{\max})$" [1]. Here, a generalized Kolmogorov's law for inertial sub-range is used to represent the intermittency and also to derive a new formulation for eddy diffusivity.

Many approaches in turbulence start assuming Reynolds' hypothesis, where the turbulence is described as a sum of a mean stream plus a fluctuation term (with zero mean). The turbulence contribution in momentum, energy, and mass equations is constituted by the product between fluctuations. These terms represent new unknowns in the equations. The system of equations can be closed using the K-theory, where the turbulent fluxes are represented by the gradient of the mean stream multiplied by an eddy diffusivity. In this paper new formulations for these parameters are derived based on the Taylor's statistical theory of turbulence [3] and by an analytical model for the energy spectra. This model of turbulence can be applied for many physical systems, such as combustion, solar physics, pollutant diffusion, and geophysical fluid dynamics.

Our approach is applied to the atmospheric turbulence, since it is a permanent feature in the planetary boundary layer (PBL), a thin layer in direct contact with the ground. In a recent paper, a similar formulation was used for convective atmospheric boundary layer to derive eddy diffusivities and counter-gradient term [4]. In the present work the intermittency is represented in a parameterization for stable boundary layer (SBL). In this PBL, turbulence comes from a delicate balance between heat flux from the atmosphere to the ground (weakening the turbulence) and the shear of the wind (production term of the turbulence).

2. Representing intermittency in the eddy diffusivity

The starting point for the new formula for the eddy diffusivity is to consider a modified or generalized Kolmogorov's energy spectra in the inertial subrange

$$E(k) = c_2\, \varepsilon^{2/3}\, k^{-(1+\zeta_2)} \tag{1}$$

c_2 being a constant, ε is the dissipation function, and k is the wavelength. For $\zeta_2 = \frac{2}{3}$ the Kolmogorov's law is recovered and there is no intermittency.

Taylor's statistical theory on turbulence [3] states that the variance of the position of a particle is related to the velocity variance according to

$$\sigma_\alpha^2 = 2\sigma_i^2 \int_0^t (t-\tau)\, \varrho_{L_i}(\tau)\, \mathrm{d}\tau \quad \text{with} \quad \begin{cases} \alpha = x, y, z\,, \\ i = u,\ v,\ w\,, \end{cases} \tag{2}$$

where ϱ_{L_i} is the correlation coefficient and satisfies $\varrho_{L_i}(0) = 1$, and the subscript L is a reference to Lagragian correlations. Assuming stationary behaviour for turbulent velocity field, the relation between the correlation function and the spectra is simplified:

$$\sigma_i \varrho_{L_i}(\tau) = \int_0^{\infty} \Phi_{L_i}(\omega) \cos \omega\tau \, \mathrm{d}\omega \,, \tag{3}$$

where $\Phi_{L_i}(\omega) = (1/\pi) \int_{-\infty}^{+\infty} \sigma_i^2 \varrho_{L_i}(\tau) \, \mathrm{e}^{\mathrm{i}\omega\tau} \, \mathrm{d}\tau$. Therefore, velocity variances can be obtained, for a frequency $n = \omega/2\pi$ in Hertz, taking $\tau = 0$:

$$\sigma_i^2 = 2\pi \int_0^{\infty} \Phi_{L_i}(2\pi n) \, \mathrm{d}n = \int_0^{\infty} S_{L_i}(n) \, \mathrm{d}n \,. \tag{4}$$

Eq. (4) shows that the kinetic energy per unit of mass is obtained if the spectrum is integrated over all frequencies, and it also introduces the spectral density $S_{L_i}(n) = 2\pi\Phi_{L_i}(2\pi n)$.

In the first-order closure, turbulent fluxes are parameterized by the product between an eddy diffusivity and a mean of the property

$$\overline{v_i' \chi'} = K_{\alpha\alpha} \frac{\partial \bar{\chi}}{\partial \alpha} \,, \tag{5}$$

where χ can be velocity, temperature and so on. An expression for the time-dependent eddy diffusivity was derived by Batchelor [5],

$$K_{\alpha\alpha} = \frac{1}{2} \frac{\mathrm{d}\sigma_\alpha^2}{\mathrm{d}t} \,. \tag{6}$$

Substituting the expression of the correlation coefficient (3) and the Taylor's equation (2) in Eq. (6) yields [6,7]

$$K_{\alpha\alpha} = \frac{\sigma_i^2}{2\pi} \int_0^{\infty} F_{L_i}(n) \frac{\sin(2\pi n t)}{n} \, \mathrm{d}n \,, \tag{7}$$

where $F_{L_i}(n) = S_{L_i}(n)/\sigma_i^2$ is the normalized Lagrangian spectra of energy. Hence, having an expression for the spectra, in which the ones matching with the generalized Kolmogorov law (1) for large values of frequency, will be a turbulence model with the intermittency represented.

The method can be applied in plasma physics, fluid flow, oceanography, and so on. In this paper the scheme will be used to derive an eddy diffusivity in atmospheric turbulence for SBL condition. Before, two aspects need to be considered in our derivation. In meteorology the Lagrangian statistics is not measured, in general, only Eulerian statistics are used. A standard procedure is to consider Lagrangian and Eulerian autocorrelation functions as similar in shape, but they are displaced by a scale factor β_i, the ratio between Langrangian and Eulerian time-scales, i.e., $\varrho_{L_i}(\beta_i \tau) = \varrho(\tau)$ [9].

The use of scale factor β_i and an asymptotic formula for large travel times ($t \to \infty$), yield to obtain the eddy diffusivity as follows [6,7]:

$$K_{\alpha\alpha} = \frac{\sigma_i^2 \beta_i F_i(0)}{4} \,, \tag{8}$$

where the scale factor can be expressed as $\beta_i = (\sqrt{\pi}\, U)/(4\sigma_i)$ [10], U being the velocity of the main stream.

A formula for spectra can be derived considering a general model, as suggested by Sorbjan [8] (see also Ref. [6])

$$\frac{n\,S_i^E(n)}{u_*^2} = \frac{A f^{m_3}}{(1 + B f^{m_1})^{m_2}} , \tag{9}$$

with $f = nz/U$ being the nondimensional frequency. Therefore, a generalized expression for the energy spectra is obtained matching the experimental spectral peak with the maximum of the spectral model, and also to fit this model with modified Kolmogorov's energy spectra. From this, the generalized expression for the normalized energy spectra is written as

$$F_i(n) = \frac{S_i(n)}{\sigma_i^2} = c_\zeta\,\zeta_2^{-1/(1+\zeta_2)} \left(\frac{f}{\rho}\right) \left[1 + \frac{1}{\zeta_2\,(f_m)_{n,i}^{1+\zeta_2}} \left(\frac{f}{\rho}\right)^{1+\zeta_2}\right]^{-1} , \tag{10}$$

where $c_\zeta = [\sin(\pi/(1+\zeta_2))]/[\pi/(1+\zeta_2)]$; and $\rho = (f_m)_i\,(f_m)_{n,i}^{-1} = 1 + 3.7\,(z/\Lambda)$ is a stability function [8,6], being $(f_m)_i$ the maximum value of the spectra and $(f_m)_{n,i}$ the maximum frequency for neutral stratification, z is the height above the surface; Λ is the local Monin–Obukhov length, obtained from the local similarity theory [11]:

$$\frac{U_*}{u_*} = \left(1 - \frac{z}{h}\right)^{\alpha_1} , \tag{11}$$

$$\frac{\overline{w'\,\theta'}}{(\overline{w'\,\theta'})_{z=0}} = \left(1 - \frac{z}{h}\right)^{\alpha_2} , \tag{12}$$

$$\frac{\Lambda}{L_{MO}} = \left(1 - \frac{z}{h}\right)^{(3\alpha_1/2)-\alpha_2} , \tag{13}$$

U_* being the local friction velocity, u_* the friction velocity on the ground, h is the SBL height, θ the temperature, and α_1, α_2 are experimental constants. The parameter L_{MO} is the Monin–Obukhov length from classical similarity theory [12],

$$L_{MO} = -\frac{u_*^3}{\kappa\,(g/\Theta_0)\,(\overline{w'\,\theta'})_{z=0}} , \tag{14}$$

where $\kappa = 0.4$ is the von Kármán constant, g is the gravity acceleration, and Θ_0 is the temperature on the ground.

From Eqs. (8) and (10), the expression for eddy diffusivity follows

$$K_{\alpha\alpha} = \frac{0.11\,U_*\,z}{(f_m)_i}\,(z\,\rho)^{-1/3+\zeta_2/2} . \tag{15}$$

The vertical component of the diffusivity tensor can be written taking the expressions for the local friction velocity U_* and for the maximum value for the vertical spectrum $(f_m)_w$:

$$\frac{K_{zz}}{u_* h} = \frac{0.32\,(1 - z/h)^{\alpha_1/2}\,(z/h)}{1 + 3.7\,(z/\Lambda)}\,(z\,\rho)^{-1/3+\zeta_2/2}\,. \tag{16}$$

2.1. Determining the scaling exponent ζ_2

It is necessary to estimate the value of the scaling exponent ζ_2, for closing our theoretical derivation. As mentioned, some theoretical objections [1,13,14] and experimental results [1,15,16] have led to different proposals of the Kolmogorov's theory. The log-normal approach is one of the first proposals [17], and the log-Poisson model [18] is another one. Here, a multifractal model will be employed.

The properties of turbulent fluxes are studied from statistics of the velocity gradient $v_r(x) = v(x + r) - v(x)$, for several scales r. From the multifractal formulation, the structure function $R_p(r)$ for inertial subrange ($\eta \ll r \ll L$) can be expressed as [1]

$$\frac{R_p(r)}{v_0^p} \equiv \frac{\langle v_r^p \rangle}{v_0^p} \sim \int_I \mathrm{d}\mu(b) \left(\frac{r}{L}\right)^{pb+3-D_F(b)}\,, \tag{17}$$

where L and η are the integral and Kolmogorov's scales, $\langle \cdot \rangle$ is the mean value, and $D_F(s)$ is the fractal dimension. In this subrange the smallest exponent will dominate expression (17), and $\zeta_p = \inf\{pb + 3 - D_F(b)\}$ is the exponent for scaling the structure function.

The fractal dimension in Eq. (17) can be computed for experimental or numerical data [1, p. 145]. However, Ramos et al. [19] derived an analytical formula for the fractal dimension based on Tsallis' non-extensive thermostatistics [20]:

$$D_F = 3\,\frac{\log\{[3 - q(r)]\,r/\eta\}}{\log(2r/\eta)}\,. \tag{18}$$

The parameter-q plays a central role in non-extensive thermostatistics, where for $q = 1$ Boltzmann–Gibbs extensive description emerges as a particular case. The parameter-q is given by

$$q(r) = \frac{15 - 7H_L(r/L)^\alpha}{9 - 5H_L(r/L)^\alpha} = \frac{15 - 7H_\eta(r/\eta)^\alpha}{9 - 5H_\eta(r/\eta)^\alpha} \tag{19}$$

with $\alpha = \zeta_4 - 2\zeta_2$, and $H_L = H_\eta\,(L/\eta)^\alpha = 3$.

3. Final comments

A new formulation for eddy diffusivity was derived for fluxes where the turbulence is generated by mechanical process in the presence of weakened turbulence. The new eddy diffusivity has a numerical value greater than that computed with Kolmogorov's hypothesis. To illustrate this feature, the vertical component of the turbulent diffusivity tensor is shown in Fig. 1, where the following numerical values

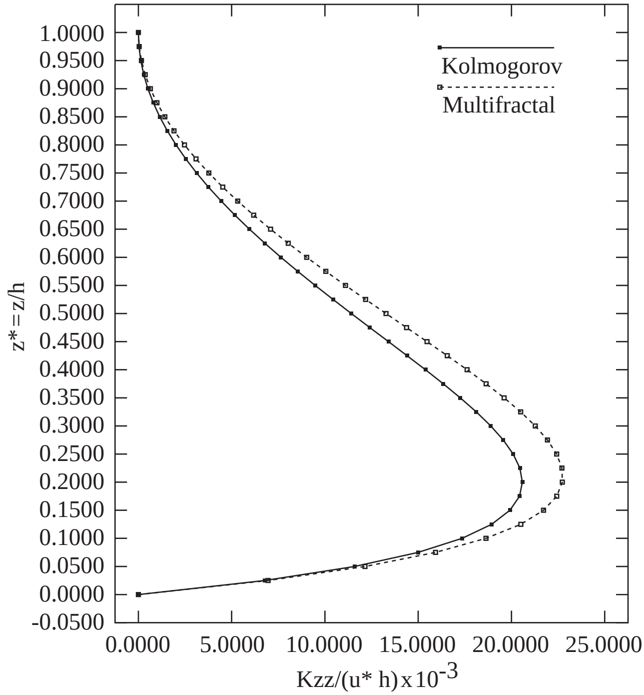

Fig. 1. Vertical eddy diffusivity for Kolmogorov's and multifractal approaches for SBL.

were used: $\alpha_1 = 3/2$ and $\alpha_2 = 1$ for fully developed SBL [11]; $(f_m)_{n,w} = 0.33$ [8], and $L_{MO} = 60\,\mathrm{m}$ as a typical value for Monin–Obukhov length in SBL; and adopting the exponent $\zeta_2 = 0.72$, such as determined by Ramos et al. [19,21]—see also Refs. [22,23] for estimation of this exponent based on non-extensive thermostatistics.

Fig. 1 shows the vertical eddy diffusivities for the Kolmogorov's original hypothesis and for expression (16). The difference between these two plots is due to the new term $(z\,\rho)^{1/40}$. These two diffusivities present a small difference, implying in a difficulty to identify an improvement of the meteorological fields predicted for the correction proposed, from regular measurements in the SBL. The same comment is valid for the attempt to identify the eddy diffusivity from observations in an SBL by an inverse problem methodology [24]. Also, it is worthy to point out that as the accuracy of observations gets improved, the effectiveness of candidate theories for estimation of ζ_2 can be evaluated, e.g., non-extensive thermostatics formalism.

Finally, it is important to mention that the result of this paper and the expression for the convective atmospheric boundary layer [4], give a new framework for two important thermodynamic types of the PBL representing the intermittency in turbulent fluxes.

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