Chapter XIV

ATMOSPHERIC VORTICES

ROGER A. PIELKE, JOSEPH L. EASTMAN, LEWIS D. GRASSO
JOHN B. KNOWLES, MELVILLE E. NICHOLLS
ROBERT L. WALKO, and XUBIN ZENG
Colorado State University
Fort Collins, CO 80523
U.S.A.

XIV.1 Introduction

This chapter discusses atmospheric vortices. §2 of the chapter presents a brief derivation of the mathematical formulation used in characterizing atmospheric vortices. Atmospheric turbulent vortices are discussed in §3. Solenoidally-generated vortices, which include sea and land breezes, mountain-valley circulations, and microbursts are discussed in §4, while small-scale vortices generated by vertical shear of the horizontal wind are briefly described in §5. Synoptic-scale vortices are introduced in §6, cumulonimbus-caused mesoscale vortical motion is presented in §7, and mesoscale vortices resulting from horizontal shear of the horizontal wind are discussed in §8. A relatively in-depth summary of tornadoes and tropical cyclones is given in §9 and §10, respectively. Finally, a summary of the chapter is presented in §11.

XIV.2 Derivation of Equations; Definitions

A convenient mathematical description of atmospheric vortices uses the vorticity equation. A general form of this equation, derived in Pielke (1984) from the momentum equation, is:

\[
\frac{\partial \rho_0 u_i}{\partial t} = -\frac{\partial}{\partial x_j} \rho_0 u_j u_i - \frac{\partial}{\partial x_j} \rho_0 \frac{u_j''}{\partial x_i} - \frac{\partial p'}{\partial y} - \left[ \frac{\partial p_0}{\partial x} \delta_{i1} + \frac{\partial p_0}{\partial y} \delta_{i2} \right] \\
+ \frac{\alpha'}{c_0^2} g \delta_{i3} - 2 \epsilon_{ijk} \Omega_j u_k \rho_0
\]  

(14.2.1)

where \( \mathbf{u}_i \) is the velocity vector, \( p \) is the pressure, \( \alpha \) is the specific volume defined as \( 1/\rho \) where \( \rho \) is the density of air, and \( \Omega_j \) is the rotation vector of the Earth. The velocities in (14.2.1) are velocities in the rotating Earth frame of reference. The independent variables are \( x_i = x, y, z \) and \( t \) where \( x \) is the east-west direction, \( y \) is the north-south direction, \( z \) is the vertical direction, and \( t \) is time.

The following three approximations have been used in the derivation of (14.2.1):

(i) Density fluctuations are small, so \( |\alpha'/\alpha_0| \) and \( |\alpha''/\alpha_0| \) are much less than unity. This assumption is referred to as the Boussinesq approximation. The conservation of fluid mass can then be written as \( \partial (\rho_0 u_j)/\partial x_j = 0 \). This approximation is valid for small Mach numbers (i.e., when the wind speeds are much smaller than the speed of sound) in the Earth's atmosphere. Variations of density for these small Mach number flows are represented by the ideal gas relationship.

(ii) The laminar molecular dissipation of velocity (i.e., \( \nu \partial^2 u_i/\partial x_j \partial x_j \)) is ignored because we will only be discussing turbulent atmospheric flows in this chapter. In turbulent atmospheric flows, the turbulent stresses (represented by the second term on the right side of Equation (14.2.1)) are much larger than the viscous stresses. Viscous effects are, however, important close to the surface of leaves, cloud droplets, etc.

(iii) The averaged quantities, denoted by the overbar, change much more slowly in time and space than the perturbations. This requirement is referred to as the Reynolds' averaging assumption. Pielke (1984, Figure 4-2) illustrates when this assumption is valid and when it is not.

Using Equation (14.2.1), a vorticity equation can be derived by applying the vector curl operator \( (\epsilon_{pqj}\partial/\partial x_q) \) such that:

\[
\frac{\partial \omega_p}{\partial t} = -\frac{\partial}{\partial x_j} u_j \omega_p - \epsilon_{pqj} \frac{\partial}{\partial x_j} \left( \rho_0 u_i \frac{\partial u_i}{\partial x_q} \right) - \epsilon_{pqj} \frac{\partial}{\partial x_j} \frac{\partial}{\partial x_j} \left( \rho_0 u_i u_i'' \right)
\]

\[
+ \delta_{i3g} \epsilon_{pqj} \frac{\partial}{\partial x_q} \alpha' \frac{\partial}{\partial x_3} \omega_p - 2\epsilon_{pqj} \epsilon_{ijk} \Omega_j \frac{\partial}{\partial x_q} u_k
\]

where \( \omega_p = \epsilon_{pqj} \partial (\rho_0 u_i)/\partial x_q \) is the density-weighted vorticity.

The individual terms in (14.2.2) correspond to:

(i) the local time derivative of \( \omega_p \)

(ii) the gradient of the grid resolvable flux of \( \omega_p \)

(iii) the mechanism to transfer \( \omega_p \) between the three spatial components as a result of velocity shear (referred to as the tilting term; it should more appropriately be referred to as simultaneous tilting and vortex line convergence. Convergence
occurs when the cross sectional area normal to the axis of the vortex tube changes.)

(iv) the creation or destruction of $\omega_p$ by subgrid-scale velocity fluxes

(v) the generation or removal of $\omega_p$ as a result of gradients in density (referred to as the solenoidal term), and

(vi) the creation or destruction of $\omega_p$ due to non-zero velocities on the rotating Earth.

Equation (14.2.2) (or alternatively (14.2.1)) represents three component equations. In addition to these relations, several other conservation equations and constraints must be simultaneously satisfied. These are (as discussed in detail in Pielke (1984)), the conservation of heat, based on the first law of thermodynamics for an ideal gas, the conservation of moisture in its three phases, the conservation of mass of air, and the conservation of other gaseous and aerosol materials in the atmosphere.

XIV.3 Three-Dimensional Atmospheric Vortices – Atmospheric Turbulent Vortices

There are several textbooks that discuss atmospheric turbulence. These include those of Lumley and Panofsky (1964) and Garratt (1992). Atmospheric turbulence is generally defined as having three-dimensional structure and consisting of a range of spatial scales of chaotic motion.

Equation (14.2.1) can be manipulated, in a manner analogous to the derivation of the Reynolds stress equations, to produce:

$$\frac{\partial \rho_0 \bar{e}}{\partial t} = - \frac{\partial}{\partial x_i} \rho_0 \bar{u}_j \bar{e} - \frac{\partial}{\partial x_j} \rho_0 \bar{u}_j \bar{u}_i \bar{e} - \rho_0 \bar{u}_j \bar{u}_i \frac{\partial \bar{u}_j}{\partial x_i}$$

$$- \frac{\partial}{\partial x_i} \rho_0 \bar{u}_j \bar{u}_i \bar{p}'' + g \frac{u''_{ij}}{\alpha_0^2 - \delta_{i3}}$$

(14.3.1)

where the Coriolis term has been dropped since scale analysis indicates that this term is negligible for the spatial scales characteristic of three-dimensional turbulence. The variable $\bar{e} = u''^2 / 2$ can be interpreted as the turbulent kinetic energy, if the averaging volume defined by the overbar is small enough such that the smaller-scale motions are three-dimensional and chaotic.

Term (i) in (14.3.1) is the flux of $\bar{e}$ by the average wind, and term (ii) is the flux of $\bar{e}$ by the turbulent wind and is referred to as a third-order correlation term since $\bar{e} u''_j = u''_j u''_i u''_i$. Term (iii) represents the shear production of $\bar{e}$. Term (iv) is the pressure correlation term which, as discussed by Lumley and Panofsky (1964),
acts to transfer kinetic energy between the three velocity components. Term (v) is a buoyancy term that can either produce or suppress turbulence.

The ratio of term (v) to term (iii),

$$R_f = \left[ \frac{\left( g w'' \alpha'' \right)}{\alpha_0^2} \right] / \left[ \rho_0 \left( \alpha_j'' \alpha_i'' \right) \frac{\partial \bar{u}_i}{\partial x_j} \right]$$

is a form of the flux Richardson number. $R_f < 0$ corresponds to the situation where buoyancy produces turbulence, while $R_f > 0$ represents the case where buoyancy suppresses the turbulence created by shear of the mean velocity. When $R_f > 0$, some of the kinetic energy caused by the velocity shear generates gravity waves, rather than turbulence.

Turbulence, resulting in vortical motion, is also generated in a saturated atmosphere if the latent heat released during condensation or deposition results in a layer in the atmosphere, or a parcel of air, which is warmer than the air in its vicinity. The atmosphere in which this vertical instability can occur is referred to as conditionally unstable (Air Weather Service Manual Technical Report-79/006 1990). Cumulus clouds are the visual manifestation when this instability is realized. The turbulent vortical structure in such clouds is particularly obvious in time lapse photography. Recently, the concept of slantwise instability (also referred to as symmetric instability) has been introduced to describe the generation of cumulus convective bands along sloped ascent (Emanuel 1983).

Atmospheric turbulence, with its large Reynolds numbers, can be considered to consist of three regions. The turbulence production region typically occurs on the larger spatial scale where velocity shear (term (iii) in (14.3.1)) and/or buoyancy (term (v) in (14.3.1)) provide the mechanism to transfer kinetic energy from the large-scale flow field to the smaller-scale turbulent fluctuations. The turbulence is dissipated at the very small spatial scales where viscous effects become dominant (for the atmosphere this scale is on the order of millimetres; see Lumley and Panofsky (1964) p.82). At the intermediate spatial scales where turbulence is neither generated nor eliminated, the kinetic energy is transferred to smaller scales through terms (ii) and (iv) in (14.3.1). These spatial scales are referred to as the inertial subrange (Lumley and Panofsky 1964). In this subrange, the kinetic energy spectrum has a $k^{-5/3}$ relationship (as predicted by Kolmogorov), where $k$ is the wavenumber of the turbulent eddy.

**XIV.4 Solenoidally-Generated Vortices**

The solenoidal term in (14.2.2) can be utilized to describe the formation of one type of vertically oriented circulation, which is referred to as a thermally forced atmospheric system. As illustrated in Figure 14.4.1, dense air adjacent to less dense air at the same pressure level will result in a flow from the denser atmosphere towards the less dense region. Since density is inversely related to air temperature
through the ideal gas law (i.e., \( T = p/\rho R \), where \( R \) is the gas constant of air), air tends to flow from cold air to warm air as a result of the solenoidal term.

Examples of these surface thermally forced atmospheric vortices include land- and sea-breezes (see Figure 14.4.2a), urban-rural wind flows, mountain-valley circulations (see Figure 14.4.3a), as well as circulations that develop in response to other spatial variations in landscape (e.g., irrigated land adjacent to dryland prairie, snow cover next to snow-free land, etc. See discussions in Atkinson (1981), Pielke (1984), and Lyons et al. (1993).
Figure 14.4.2 A simulated sea breeze of the flow in the plane of the figure for (a) zero synoptic flow ($V = 0 \text{ m/s}$), (b) an onshore large-scale wind of 3 m/s ($V = -3 \text{ m/s}$).
Figure 14.4.2 A simulated sea breeze of the flow in the plane of the figure for (c) an offshore large-scale wind of 3 m/s \((V = 3 \text{ m/s})\), and (d) a strong offshore wind of 6 m/s \((V = 6 \text{ m/s})\). The time of the cross sections are 1800 LST and the land is shown by the black strip. The solid line denotes the potential temperature, and the dotted line denotes the horizontal velocity.
Figure 14.4.3 A simulated mountain-valley circulation (a) in the absence of large-scale flow ($V = 0 \text{ m/s}$) and (b) in the presence of a 3 m/s large-scale flow from the west ($V = 3 \text{ m/s}$). The solid line denotes the horizontal velocity, and the arrows denote the horizontal and vertical velocity vectors.
When a large-scale lower tropospheric flow is superimposed on this solenoidal circulation, the horizontal density gradient is strengthened locally where the solenoidal flow opposes the larger-scale wind. This phenomenon can result in a stronger vortex, as long as the larger-scale wind is not too strong. Estoque (1962) originally reported this vortex strengthening for the sea breeze problem (see Figures 14.4.2c and 14.4.2d). This interaction of the large-scale wind and the solenoidal circulation is a nonlinear process, and not a simple linear superposition of the two flows. An onshore large-scale wind tends to weaken the solenoidal circulation, as illustrated in Figure 14.4.2b and discussed in detail by Arritt (1989). Figure 14.4.3b illustrates the vortex strengthening in the context of the mountain-valley circulation.

Solenoidal circulations can also develop from non-surface forcing. For example, cold downdrafts from thunderstorms or from larger-scale cold advection will spread laterally when the air reaches the surface. The result is the generation of horizontal vorticity associated with the leading edge of this cold gust front (Shapiro 1984).

When the winds and change of winds with time and space are sufficiently large, thunderstorm downdrafts are referred to as downbursts (or, alternatively, microbursts) as discussed in McCarthy et al. (1982) and Wilson et al. (1984). Microbursts have been implicated in catastrophic airliner crashes (Pielke 1990).

The boundary between air mass type also is often associated with solenoidal circulations which are modified by the Earth’s rotation. This type of circulation is referred to as a frontal circulation and is discussed in detail in Williams (1972), Hoskins and Sharp (1982), and others.

**XIV.5 Small-Scale Vortices Generated by Vertical Shear of the Horizontal Wind**

Vortices can develop as a result of other forcings in the density-weighted vorticity equation. For instance, breaking waves in the vertical direction (referred to as Kelvin-Helmholtz instability) occur when the generation of vorticity by vertical wind shear interactions (term (iii) in Equation (14.2.2)), is more rapid than the damping of the wave by the buoyancy term (term (v)). Airflow over terrain irregularities can also develop closed circulations as waves are excited by the forced vertical displacement of the air which can result in overturning vortical motions if the displacement is sufficiently large (Smith (1979) has a discussion of this type of circulation).

**XIV.6 Synoptic-Scale Vortices**

There has been considerable study of large-scale horizontal vortices in atmospheric science (Carlson 1991; Bluestein 1992; and Holton 1992). Having spatial scales of hundreds to thousands of kilometres, these synoptic-scale meteorological fea-
tasures include extratropical and tropical cyclones, polar and subtropical anticyclones (Pielke et al. 1987), and polar lows (Montgomery and Farrell 1992). As a result of the influence of the Earth’s rotation (i.e., the Coriolis force), these circulations tend to rotate counterclockwise around low pressure systems and clockwise around high pressure systems, in the northern hemisphere, when viewed from above. Figure 14.6.1a presents a geostationary earth orbiting satellite (GOES) image of upper tropospheric water vapor and Figure 14.6.1b shows a GOES infrared image of cloud systems, to illustrate these large-scale vortex motions.

There are several flow instabilities that generate these vortices. Inertial instability was first defined as centrifugal instability (Emanuel 1984). In a classical two-dimensional inertial stability problem, if the angular momentum decreases with the radius in an axisymmetric vortex, it is inertially unstable. In the three-dimensional case using a rotating coordinate system, inertial instability occurs when there exists a region of absolute vorticity of opposite sign to the large-scale vorticity such that an imbalance occurs between the horizontal pressure gradient force and the Coriolis force. For zonal conditions, as shown by Holton (1979), the inertial instability condition in the northern hemisphere is $f - \frac{\partial U}{\partial y} < 0$, where $U$ is the large-scale zonal wind and $f = 2\Omega \sin \phi$. The independent variable $y$ is the north-south direction, and $\phi$ is the latitude. Such an instability often occurs as air flows around high pressure systems and the pressure gradient force is not large enough to balance the outward centripetal acceleration and the Coriolis effect. Ciesielski et al. (1989) illustrate from satellite imagery the generation of unstable, relatively small-scale horizontal vortices associated with the anticyclonic side of a strong upper tropospheric subtropical jet stream as it flows around the northside of a subtropical high pressure system in the northern hemisphere.

Barotropic instability resulting in vortical motion develops as a result of large horizontal gradients of velocity. Holton (1992) indicates that a necessary condition for this instability is that the horizontal gradient of absolute vorticity must change sign somewhere in the region. For zonal flow conditions, the barotropic instability condition in the northern hemisphere is

$$\frac{\partial f}{\partial y} - \frac{\partial^2 U}{\partial y^2} \leq 0.$$
Figure 14.6.1 (a) GOES satellite image of upper tropospheric water vapor for 15 February 1992 at 0341 Z, and (b) GOES infrared image of cloud systems for 15 February 1992 at 0401 Z.
exponentially with the separation distance between the two PV regions (Hoskins et al. 1985).

**Baroclinic instability** occurs as a result of the interaction of temperature advection superimposed on a velocity field. Charney (1947) introduced this concept to describe the conversion of potential energy to kinetic energy characteristic of mid- and high-latitude extratropical cyclonic vortices. Since the horizontal temperature gradient is largest near the polar front that demarcates air of polar origin from that of tropical origin, these cyclonic vortices develop along this front. The vertical wind shear of the horizontal wind is also large in this region as a result of the large horizontal pressure gradient differences in the middle and upper troposphere, which are caused by the large horizontal temperature difference between the polar and tropical air throughout the troposphere. Carlson (1991) summarizes several of the characteristics associated with this instability, including that larger vertical wind shears yield greater growth rates and longer wavelengths, and that below a critical vertical wind shear (about 2000 km) no growth occurs. The optimal growth rate occurs for horizontal wavelengths of 3000 to 4000 kilometres which explains, for example, why mid-latitude synoptic storm systems that influence our day-to-day weather are typically this size. Examples of papers which analyze baroclinic instability include Bannon (1989) and Reinhold (1990).

From the potential vorticity point of view, a necessary condition for baroclinic instability is for the northward gradient of PV evaluated on a potential temperature, $\theta$, surface to change sign on a different $\theta$ surface, i.e.,

$$\left. \frac{\partial PV}{\partial y} \right|_\theta > 0 \text{ and } \left. \frac{\partial PV}{\partial y} \right|_{\theta + \Delta \theta} < 0$$

Similar to the barotropic case, these PV regions must be close enough in the vertical to influence one another, otherwise the above criterion is not sufficient. Mathematically, barotropic and baroclinic instabilities are quite similar.

### XIV.7 Cumulonimbus-Generated Mesoscale Vortices

Mid-level cyclonic vortices of an average diameter of 150-300 km can be generated by mesoscale cumulonimbus convective systems (Johnson et al. 1989; Bartels and Maddox 1991). Mesoscale convective systems in their mature stages have large stratiform anvils typically with latent heating at mid and upper levels in the tropospheric and cooling at low levels. These systems are characterized by a mid-tropospheric-level low pressure anomaly with high pressure at the surface and upper troposphere. Mid-level inflow occurs with lower- and upper-level outflows. The effect of the Coriolis force on the middle-level inflow is to generate a cyclonic vortex. These vortices can generally be identified by visible satellite imagery, where
the larger ones are referred to as mesoscale convective complexes (Cotton 1990). Durations of these vortices on satellite imagery are typically less than 12 h, unless new convection develops within the vortex which occasionally occurs. Although the vortex may no longer be identifiable from satellite imagery after the clouds have dissipated, the cyclonic circulation may persist for many hours. These vortices have been observed to develop into tropical cyclones when they move over warm oceans (Laing and Fritsch 1993).

XIV.8 Mesoscale Vortices Generated by Horizontal Shear of the Horizontal Wind

Vortices of predominantly vertical vorticity are also generated as flow passes terrain obstacles such as islands and mountains. The horizontal velocity shear resulting from an impediment to the flow can generate vortices when vertical ascent over the obstacle is inhibited by a stable atmospheric stratification. A quantity defined as the Froude number \((Fr = U/NH)\) where \(U\) is the upstream velocity, \(N\) is the Brunt Väisälä frequency, and \(H\) is the height of the obstacle) is used to estimate whether or not mesoscale vortices will be shed by the terrain obstacle as flow moves past it. Small values of \(Fr (Fr << 1)\) promote the creation of this type of circulation. Chopra (1973) discusses this type of vortex generation. Smolarkiewicz et al. (1988) describe the alteration of flow associated with the island of Hawaii as related to \(Fr\). Crook et al. (1990) discuss such vortices that result due to airflow in the complex terrain of the Colorado Front Range.

XIV.9 Tornadoes

Tornadoes, the most violent of windstorms found in the atmosphere, are intense vortices normally associated with thunderstorms (Figure 14.9.1).

They are most common in the mid-western United States in spring and early summer, and in the Gulf States in late winter and early spring, but have been observed in all seasons and in many other parts of the world as well. The strong winds of a tornado extend to the ground, where they cause considerable destruction, injury, and loss of life annually (see Grazulis (1990) for a comprehensive listing of significant tornadoes in the U.S.).

Tornadoes occur in a variety of shapes and sizes, and the flow within them can range considerably in complexity. The core of the vortex, defined as the region between the axis and the radius at which the maximum azimuthal wind component occurs, typically ranges from a few tens of metres to as much as a kilometre in diameter. By Stokes' theorem [see (1.1.3)], the component of vorticity parallel to the vortex axis is large on average in the core, although it may be non-uniformly distributed. Outside the core, axial vorticity is much weaker, and is often close to
zero. The circulation (1.1.9) around a ring centered on and perpendicular to the vortex axis is often nearly a constant function of radius outside the core, making it a useful global descriptor of a tornado. Circulation is the most influential parameter controlling core diameter (Lewellen 1976), and ranges from less than $10^4$ m$^2$/s in narrow tornadoes to more than $3 \times 10^5$ m$^2$/s in the widest tornadoes. Although a tornado axis may tilt considerably from the vertical, the tilt may be considered a variant on what is essentially an upright axis extending from the ground to a height of usually several kilometres.

The precise mechanisms responsible for producing the intense field of vertical vorticity in a tornado core remain a topic of research. However, it is now fairly well established that the process involves first a production of relatively weak vertical vorticity over horizontal distances at least several times the (future) tornado core diameter, followed by subsequent concentration of that vorticity by two-dimensional convergence in the plane orthogonal to the axis (see §I.3.4), as described by term iii of Equation (14.2.2). That is, the most intense vorticity is neither generated in place, as by solenoidal processes, nor generated horizontally at full intensity and subsequently simply tilted into the vertical. Walko (1993) discusses the vortex line configuration in a typical mature tornado, wherein vortex
lines trace paths which are oriented axially through the vortex core and spread radially outward in all directions to distances of several core diameters upon reaching the surface friction layer. He argues such a configuration to be evidence that those vortex lines pre-existed with vertical tilt and wider horizontal spacing, and were materially transported toward the tornado axis by horizontally-convergent winds.

Convergence of vortex lines acts not only to increase vorticity, but also to increase rotational velocity as well. This may be illustrated by considering the circulation \( \Gamma \) around a material ring coinciding with the core wall of a vortex. By Kelvin's Theorem (1.2.13), \( \Gamma \) will remain constant as convergence of fluid toward the vortex axis advects the ring inward and reduces its radius and circumference. By the definition of circulation (1.1.9), the velocity must increase as the path length decreases.

Given that vortex line convergence is the final process in tornadogenesis, the question of how a tornado forms translates to: (1) how are vortex lines assembled in the first place over a region at least a few core diameters wide, and (2) how is the kinematic flow field established which concentrates the vortex lines? Precise answers to these questions are less certain, and probably vary widely from case to case. Two examples which have received some attention are described in a simplified manner below. Wilczak et al. (1992) reported a fairly weak tornado in eastern Colorado which formed along a boundary between two distinct air masses. The two air masses had dissimilar mean horizontal velocity vectors, such that the dividing boundary was a region of both horizontal convergence and vertical vorticity. Upward motion resulting from the convergence carried sufficient water vapor aloft to cause condensation. The associated release of latent heat warmed the ascending air, causing it to become more buoyant and rise faster. Convergence along the boundary was likewise intensified, and the process continued to amplify. Eventually, convergence persisted sufficiently for the pre-existing vertical vorticity (vertical vortex lines) to become more concentrated, and a tornado resulted. The major impetus for vortex line convergence here was horizontal convergence beneath the convective cell, while the vertical orientation of vortex lines apparently existed prior to mature convection. The waterspout, a vortex similar to a tornado but forming over water, has been observed to commonly develop along convergence lines (Simpson et al. 1985), and may often develop by a similar mechanism.

The local environment for most strong tornadoes appears to be set up in a more complicated way (e.g., Lemon and Doswell 1979). Here, vertical vorticity does not precede convection, but results from it. In the pre-storm environment, considerable horizontal vorticity (i.e., the vertical shear of the horizontal wind) exists in, at least, the lowest 1 or 2 km of the atmosphere. Upward convective motion in the storm transports a portion of the vortex lines upward, resulting in a vertical vorticity component where the lines slope (Lilly 1982; Davies-Jones, 1984). Equivalently, upward motion transports low-level horizontal momentum to middle and upper levels of the storm where it differs from the ambient momentum outside the storm; vertical vorticity results from horizontal gradients of horizontal momentum, and
typically occupies a region approximately 10 km wide. Production of low-level vertical vorticity, which is required for tornado formation at the ground, appears to require localized downward motion in order to transport middle-level horizontal momentum to a limited region of the surface, a form of tilting horizontal vorticity into the vertical (described by term iii of Equation 14.2.2). This was argued on theoretical grounds by Davies-Jones (1982), and is supported by the observation that many tornadoes develop only after a nearby precipitation-induced downdraft forms. The horizontal vorticity itself may be primarily that which existed in the pre-storm environment (Davies-Jones 1993; Walko 1993), or may be generated solenoidally, for example by evaporative cooling of rain (Rotunno and Klemp 1985). This second mechanism of tornado formation differs from the first in that the low-level vertical vorticity was produced by the storm itself, or by the interaction of the storm with its environment, rather than existing before the storm. The mechanism for concentrating that vorticity in both cases, however, is the low-level horizontal convergence caused by the convective updraft.

In an example of tornadogenesis provided by numerical simulation (Grasso 1992), a significant low pressure region with vertical continuity was found to form in the large horizontal gradients of vertical motion at low to mid levels of the storm (see Figure 14.9.2). The interaction of the three-dimensional vorticity field with the updraft allows the low pressure to form at lower levels of the storm, but still above cloud base. In time, a low pressure tube exists from low to mid levels to cloud base. The existence of the low at cloud base, and still in the updraft gradient, locally enhances upward vertical motion. This in turn enhances, via continuity, horizontal convergence of the subcloud air. Due to the enhanced horizontal convergence, pre-existing vertical oriented vortex lines converge beneath the low. This results in the three-dimensional vorticity vector essentially oriented vertically upward, which grows in magnitude thus allowing further descent of the low pressure tube beneath cloud base. The region of low pressure now coincides with the updraft of the developing funnel as opposed to being in the horizontal updraft gradient above cloud base. The descent of the pressure tube, and thus the funnel, interacts with and responds to surface forces allowing the descent of the funnel to the ground, thus forming a tornado.

It was once thought that the background vertical component of vorticity associated with the Earth's rotation could be concentrated into a tornado vortex by the horizontal convergence beneath a sufficiently long-lived convective updraft, but that vorticity is now generally regarded as too weak given the time available. Nevertheless, the Earth's rotation is responsible, though less directly, for establishing that approximately 90% of tornadoes rotate in the same sense as the Earth (counterclockwise in the northern hemisphere, as viewed from above). The Earth's rotation establishes an ambient wind pattern, in a typical tornado-prone environment, in which the vector of vertical shear of the horizontal wind rotates clockwise, in the lowest few kilometres of the atmosphere. This in turn favors development of counterclockwise rotation of the air in which the tornado forms (Rotunno and
Figure 14.9.2 Horizontal cross section through a thunderstorm at 1633 m and 99 min into a tornado simulation. Solid contours depict upward motion in m/s, long dashed lines show the perturbation pressure deficit in Pa. Positive vertical vorticity is depicted by dash-dot lines and negative vertical vorticity by short dashed lines in s⁻¹, and horizontal vorticity vectors by arrows. The maximum vector is shown in the lower right of the figure (Grasso 1992).
Klemp 1982; Lilly 1982). Lilly (1986) introduced the concept of helicity, which is the vector inner product of velocity and vorticity to characterize the occurrence of large rotating thunderstorms.

It is possible for mechanisms other than a convective updraft to cause low-level convergence. For example, a precipitation-cooled downdraft can spread laterally upon reaching the ground, and the leading edge of the advancing cold air can produce a region of horizontal convergence, capable of concentrating vertical vorticity. Tornado-like vortices are sometimes observed along these convergence boundaries. Such convergence is far weaker than that fostered by deep convection, however, and is insufficient for maintaining any but weak vortices.

The maximum wind speeds commonly attained in tornadoes, and the maximum theoretically attainable wind speeds, are both subjects of considerable interest and controversy. Historical estimates have placed winds as fast as the speed of sound, which would require a nearly complete vacuum at the vortex axis to provide the necessary centripetal force. Few, if any, still believe such high estimates, and values in the range of 100 to 150 m/s are most commonly cited for the strongest of tornadoes. Measurements of tornado winds are scant, but have yielded some useful information. Analyses of motion pictures of tornadoes and of ground marks left by tornadoes (Fujita et al. 1976), studies of damage to man-made structures (Mehta 1976), and direct Doppler radar measurements of tornado winds (Bluestein and Unrath 1993) generally yield estimates in or below this range (see also a review of tornado wind speed estimates by Golden (1976)). Nevertheless, each of these methods has the potential of failing to detect the actual highest wind speed within a tornado, which may be highly transient and localized, may occur a considerable height above the ground, and may involve vertical motion not detectable in a nearly horizontal radar beam. While no reliable measurement has indicated tornadic wind speeds to substantially exceed 150 m/s, there still exists no proof that such winds are impossible in normal tornadic situations.

Methods for theoretically estimating tornado winds generally begin from simplified models which assume the vortex to be axisymmetric with a vertical axis, and consider more general vortex structures as deviations from the simple model. The simplest model is based on the assumption that the tornado vortex extends from the ground up to the top of a deep convective cloud, which is warmer than its environment due to latent heating from condensation. The model assumes that the pressure deficit on the vortex axis at the ground, relative to the ambient ground pressure well outside the tornado, is determined by the warmer air along the axis and the resultant reduction of the weight of the air over this location. The rotating vortex winds are then deduced from the pressure deficit through dynamic relationships. This simple model, however, falls short of explaining the most intense tornadic winds.

Several modifications to the simple model have been proposed to yield estimates closer to the maximum observed winds. Lilly (1969) hypothesized that the vortex core may be even warmer than possible by latent heat release as a result of dry
air from above the cloud being drawn downward into the low-pressure vortex core. Such subsidence would result in higher parcel temperatures due to adiabatic compression, and is responsible for the excessively warm temperatures occurring in hurricane eyes. Walko (1988a) tested this hypothesis with an axisymmetric numerical model, and found that for vortices having circulations comparable to the largest observed in tornadoes, sufficient axial subsidence can indeed occur to produce the higher observed tornado wind speeds.

Terms neglected in the simple model are likewise capable of augmenting the axial pressure deficit, and hence the maximum winds in a tornado. Lower surface pressure can occur if air along the axis is accelerating downward. Walko (1988a) obtained numerical solutions with oscillatory vertical motions along the axis, which during the downward acceleration phase contributed significantly to the surface pressure deficit. Turbulent stress also contributes toward lower surface pressure if it is directed upward along the axis. This situation occurs when air adjacent to the axis rises faster than air along the axis, which is sometimes the case (Ward 1972; Baker and Church 1979). The magnitude of this term has been large in numerical simulations, but its importance in actual tornadoes is unknown. Vertical kinetic energy contributes to lower pressure on the axis at any height above the ground where vertical motion is nonzero. Particularly strong upward motion along the axis sometimes occurs because of drag forces on the vortex by the lower boundary. The drag reduces rotational speed weakening the centripetal acceleration. The radial pressure gradient is thus underbalanced by the centripetal acceleration, and inward acceleration results in a shallow region called the vortex boundary layer. The enhanced inflow of air converges on the axis and is forced to turn upward along the axis by continuity. Under conditions where the inward flux of air in the boundary layer is large and the vortex core is narrow, an intense upward jet of air results along the axis, leading to large values of vertical motion. A several-fold amplification of the local pressure deficit coincident with this jet, relative to the pressure deficit at the surface where vertical motion is zero, has been demonstrated in low Reynolds number laboratory vortices and numerical simulations (Wilson and Rotunno 1986; Fiedler and Rotunno 1986). Walko (1988b) presented evidence that this phenomenon should be weaker in the much higher Reynolds number flow of a tornado, but observations have strongly suggested the phenomenon to nonetheless be important in some tornadoes.

Another mechanism which may contribute significantly toward higher peak wind speeds is the occurrence of multiple vortices in some tornadoes (Fujita 1970; Fujita et al. 1976; Bienkiewicz and Dudhia 1993). Laboratory (Church et al. 1979) and numerical (Walko and Gall 1984) studies have shown that multiple vortices occur in tornadoes containing an inner cell in which air descends along the axis, turns radially outward upon nearing the ground, and returns upward in an annular region surrounding the axis. The inner cell is contained entirely within the vortex core, and the radial outflow in the cell tends to transport any axial vorticity outward. The vorticity in the core thus becomes concentrated in an annular
region near the boundary of the inner cell, a flow configuration which is unstable to axially-asymmetric perturbations. The disturbances which develop as a result form into multiple quasi-circular vortices whose axes are located along the inner cell boundary and revolve around the main tornado axis. The motion within the multiple vortices is the combined motion from the rotations of both the multiple vortices and the main tornado, and may exceed the rotational speeds of an axisymmetric tornado. It has been speculated that multiple vortices add considerably to the destructiveness of tornadoes in part because they cause wind direction to change extremely rapidly as they move past objects on the ground. From photographic evidence and damage path assessments, this higher velocity air is evident as particularly well-defined rotating air columns within the larger tornado and are referred to as suction vortices.

**XIV.10 Tropical Cyclones**

Tropical cyclones are intense vortical storms which develop over the tropical oceans (Figure 14.10.1). About 80 tropical cyclones form over the tropical oceans each year (Gray 1975). They occur in regions where the sea surface temperature exceeds 26°C. When the maximum wind speeds of tropical cyclones equal or exceed 65 knots they are called *hurricanes* in the Atlantic and east Pacific, *typhoons* in the western north Pacific, and *cyclones* in the south Pacific. In the range between 35 to 65 knots they are called *tropical storms*. In a hurricane, the average radial extent of hurricane force winds is about 100 km, but gale force winds (greater than 28 knots) may extend 500 km from the center. Extremely low surface pressure anomalies occur at the center of the vortex, which can exceed 100 mb in magnitude. The energy source for the tropical cyclone is the deep convective clouds near its center which develop over the rich water vapor and heat source of the tropical and subtropical oceans. (Palmen and Riehl 1957; Dunn and Miller 1960; Anthes 1982).

Most of the tropical cyclone is in gradient wind balance, where the centripetal and Coriolis forces nearly balance the horizontal pressure gradient force. However, surface friction eliminates this gradient wind balance in the lowest kilometre or two, causing air to spiral inward and toward low pressure. The strong cyclonic tangential circulation is produced by the inward advection of air with large absolute angular momentum from the tropical cyclone environment. As the air spirals inwards towards the center of the vortex, large sea surface fluxes of latent heat and to a lesser extent, sensible heat are important for providing the fuel for deep convection. The sensible heating from the ocean permits isothermal inflow air, rather than a cooling of air parcels due to adiabatic expansion as the air encounters the lower pressure of the tropical cyclone core. The deep cumulus convection occurs in a ring called the *eye wall*, which surrounds a region of relatively calm and clear air (referred to as the *eye*). The large azimuthal momentum the air has acquired is transported upwards in the convective towers of the eye wall, resulting in a very
deep cyclonic vortex. Strong radial outflow occurs in the upper troposphere where the air travels away from the storm, eventually acquiring an anticyclonic vortical motion as a result of the Coriolis force. In the center of the eye weak subsidence occurs leading to dry adiabatic warming. The central core of the vortex is very warm relative to the environment, with temperature anomalies larger than 10°C normally occurring in the upper troposphere of mature hurricanes. The warm core hydrostatically accounts for the low surface pressure anomaly. The surface isobars are nearly concentric circles with a minimum at the center of the storm.

Tropical cyclones usually develop from a pre-existing tropical disturbance that consists of organized cloud and wind patterns, often very similar to the mesoscale convective systems discussed in §7. Atlantic tropical disturbances, or cloud clusters, often form over the eastern Atlantic, or over central Africa, and subsequently move westward. Small amplitude easterly waves, resulting from barotropic and/or baroclinic instability, sometimes lead to the formation of cloud clusters, which may then become hurricanes (Riehl 1948, 1954; Burpee 1972). However, only about 10% of cloud clusters actually become tropical storms. Tropical cyclone "genesis" refers to the transformation of a cloud cluster into a tropical storm. The processes leading to genesis of tropical cyclones is still not well understood, but conditions

Figure 14.10.1 GOES visible satellite image of Hurricane Hugo at 1930 Z on 21 September 1989. Photo courtesy of Dr. Ray Zehr, NOAA/NESDIS, RAMM Branch, Colorado State University, Fort Collins, Colorado.
favorable for genesis have been identified. A sea surface temperature of at least 26.5°C is required (Palmén 1948), and the tropical disturbance needs to have deep convective clouds (Riehl 1954). In addition to these factors, Gray (1979) identifies the following factors as favorable for cyclogenesis: high mid-tropospheric relative humidity, large surface to 500 mb lapse rates of equivalent potential temperature, small tropospheric vertical wind shear, high magnitudes of low-level relative vertical vorticity, and sufficiently high values of the Coriolis parameter.

However, these are not sufficient conditions, and it appears that some external forcing is required. Many case studies suggest that low-level external forcing in the form of a wind speed maxima penetrating the circulation of a pre-existing disturbance may initiate cyclogenesis (e.g., Love 1985). Such features are referred to as wind surges. There may be two distinct stages where external forcing is important. A wind surge, or an externally forced increase in low level convergence, may initiate intense deep convection in a cloud cluster, which may then develop a weak mesoscale vortex (similar to those discussed in §7) and a lowering of surface pressure of 1–3 mb. A second surge, occurring 2–3 days later, acting on this pre-existing vortex may then lead to another convective maxima which strengthens the vortex into a tropical storm (Zehr (1992) discusses this scenario for tropical cyclogenesis in the western north Pacific). An element of chance is envisioned in this process, since favorable external forcing, resulting in enhanced low-level convergence, is required to act on a cloud cluster which already has a pre-existing vortex. There is also observational evidence that environmental forcing at upper levels by synoptic-scale anticyclonic vortices can play a role in cyclogenesis (Molinari 1988; Davidson et al. 1990). Montgomery and Farrell (1993) discuss the role of upper level potential vorticity disturbances on the formation of tropical cyclones.

Tropical cyclone ‘intensification’ refers to strengthening of the system once it has been designated a tropical storm. As in the case of tropical cyclogenesis, external forcing due to large-scale circulations at upper levels seems to be an important environmental forcing for tropical cyclone intensity change (Holland and Merrill 1984; Challa and Pfeffer 1990; Davidson et al. 1990). Another proposed explanation is convective forcing (Ooyama 1982), in which development occurs through a cooperative feedback between latent heat release in deep cumulus convective clouds and the developing disturbance. Emanuel (1986), and Rotunno and Emanuel (1987) emphasize the importance of ocean-atmospheric interaction, in which self-induced latent and sensible heat transfer from the ocean to the atmosphere act to spin up a finite amplitude disturbance. However, tropical cyclone intensity change remains a very challenging forecasting problem.

Spiral rainbands are a common feature of tropical cyclones. Inertial gravity waves have been proposed as a mechanism for producing rainbands (Kurihara 1976; Willoughby 1977). Recent observational studies have described the structure of tropical cyclone rainbands (Powell 1990; Ryan et al. 1992). The width of rainbands is typically 10–30 km, but can be as large as 100 km. Rainbands are often composed of cumulus convective cells intermingled with stratiform precipitation.
Since the tropical cyclone has low-level radial inflow and upper-level radial outflow, there is significant shear normal to the rainbands. The low-level inflow feeding the convective cells is mainly from the outer side of the band (farthest from the cyclone center). Precipitation induced downdrafts can modify the subcloud layer, which has led to speculation that they could reduce the energy available for convection in the eye wall (Powell 1990). The bands are also a region of azimuthal wind speed maxima. There is some evidence that their development may be influenced by the translation speed of the storm (Shapiro 1983).

The eyewall of the tropical cyclone is often disorganized when the storm is weak, but becomes more organized as it strengthens. For a hurricane, mean updrafts are ~5 m/s with maximum core updrafts of ~10 m/s. Convective cells are typically 2-5 km in diameter. The updrafts appear to slope radially outward with altitude (Jorgensen 1984; Black and Willoughby 1992). Steeper sloping updrafts appear to be associated with stronger storms (Shea and Gray 1973). Precipitation adjacent to the eyewall is predominantly stratiform. Sometimes spiral rainbands may form a partial or complete ring of heavy precipitation around the eyewall, with a well-defined wind maxima. This pattern of inner and outer convective rings is generally referred to as concentric eyewalls. Tropical cyclones with concentric eyewalls sometimes undergo a transformation in which replacement of the inner eyewall coincides with a decrease in storm intensity. Shapiro and Willoughby (1982) showed that a simple model of balanced vortex response to added heat or momentum can explain convective-ring contraction. Weakening occurs because the outer eyewall may act as a barrier to the flow of saturated air into the vortex center and, more importantly, because the outer eyewall, as it intensifies and contracts, induces strong subsidence over the inner eyewall. These two effects disrupt the transverse circulation of the inner eyewall, causing the central pressure to rise and leading to a radial wind profile that is weaker and more uniform than before the outer eyewall formed.

Numerical models have contributed significantly to our understanding of tropical cyclones and are used routinely as a forecasting tool. Early attempts to numerically simulate the tropical cyclone made use of a simplified set of equations and are described as balanced models (Ooyama 1969, 1982). They were axisymmetric and the effects of latent heating in convective clouds were not explicitly resolved, but were represented by a cumulus parameterization scheme. The simulated tropical cyclone exhibited a life cycle of growth, maturity, and decay. Experiments indicated the importance of a high sea surface temperature and the related sensible and latent heat supply from the ocean.

Three-dimensional primitive equation models were developed (Kurihara and Tuleya 1974; Mathur 1974), but they still made use of cumulus parameterization closure; an approach which significantly simplifies the physics associated with deep cumulus convection. Many of the observed features of tropical cyclones were reproduced, however, including spiral rainbands and a strongly asymmetric outflow layer. Model simulations support the view that intensification can occur as a re-
result of a cooperative feedback between latent heat release in convective clouds and the developing disturbance. The convective clouds supply the heat necessary to drive the large-scale disturbance, by producing a radial pressure gradient, and the large-scale disturbance produces the moisture convergence necessary to drive the convection. A positive feedback can ensue, which is eventually limited by the increased stability to vertical motions as the inner core warms. Models which utilize cumulus parameterization have been used to investigate mechanisms leading to the genesis of tropical cyclones (Tuleya 1988; Challa and Pfeffer 1990) and tropical cyclone motion (Kurihara et al. 1990). A drawback of these models is that a cumulus parameterization is used to provide the latent heating which drives the model vortex.

The first numerical models of tropical cyclones which attempted to resolve convective clouds were those of Yamasaki (1977), Rosenthal (1978), and Jones (1980). Although large grid increments were used (~10 km), these models still produced some realistic features. Willoughby et al. (1984) simulated a hurricane using an axisymmetric, nonhydrostatic model with horizontal grid increments of 2 km, at the center of the domain. Concentric rings of convection were obtained which contracted about the vortex center. Realistic features included: the outward slope of the eyewall updraft and tangential wind maximum; the relative location of the updraft wind maximum and precipitation maximum; stratiform precipitation and mesoscale downdrafts outside the eye; and mid-level radial inflow.

A three-dimensional convectively explicit nonhydrostatic simulation has been made by Tripoli (1993). The model used nested grids (Clark and Farley 1984) with a fine grid increment of 10 km for most of the simulation, but this was reduced to 3.3 km for a six hour period. Emanating inertia gravity waves were produced by transient mesoscale convective features as the tropical cyclone developed. It appeared that the increasingly strong inertial frequency of the core acted to increasingly trap the convection induced heating within the core by balancing the tangential wind against the low pressure. This process has been described in a theoretical study by Schubert and Hack (1982). A three-dimensional convectively explicit nested grid simulation has also been made by Nicholls and Pielke (1993). A fine grid increment of 3 km was used throughout the simulation with a fine grid domain size of 270 x 270 km². There were 27 vertical levels. The simulation was initialized with a deep vortex in gradient wind balance (maximum winds of 25 m/s). Frictional convergence led to the development of deep convection and rapid intensification. Figure 14.10.2 shows a horizontal cross section of wind vectors and total condensate at z = 2.5 km and at t = 20 h. At this level, a strong vortex is evident with tangential wind speeds of ~50 m/s. A well defined clear eye of radius 15 km is apparent, which extends throughout the depth of the storm. The structure of the simulated tropical cyclone is in good agreement with the general structure of observed systems (Figure 14.10.2).

Although convectively explicit simulations, such as this one, are computationally expensive, they are much more likely to realistically represent the effects of
Figure 14.10.2 Horizontal cross section of wind vectors and total condensate at $z = 2.5$ km and at $t = 20$ h for a hurricane simulation.
convective clouds than models which use cumulus parameterizations. They are certain to be used increasingly in the future to investigate all aspects of the life cycle of the tropical cyclone.

The Great Red Spot (GRS) is a historically fascinating vortex observed in the atmosphere of Jupiter, which has a similarity to hurricanes on the Earth. The GRS has a depth of about 200 km and is above the neutrally-stratified deep atmosphere. Horizontally, the GRS covers 20000 km in longitude so that it has a 100-to-1 ratio in horizontal to vertical dimensions. Most current models of the GRS are based on a two-level model, where a thin upper weather layer, which contains the vortex, overlies a much deeper layer, which is meant to represent the neutrally-stratified deep atmosphere (Dowling and Ingersoll (1989) and references therein). Any motions in the deep layer are assumed to be zonal and steady. This two-layer model is dynamically equivalent to a one-layer shallow water model with meridionally varying solid bottom topography. Using the GRS cloud-top velocity data from the Voyager spacecraft, Dowling and Ingersoll (1989) derived the bottom topography and they found that the large vortices in their shallow water model do not decay in the presence of dissipation. Instead, these vortices maintain themselves against dissipation by absorbing the smaller vortices, and the system achieves a dynamical equilibrium. In the work of Dowling and Ingersoll (1989), the small vortices arise spontaneously in the unstable shear flow, but a crucial unanswered question for the model and for Jupiter itself is how the cloud-top zonal velocity profile is maintained in its present unstable state. The small vortices can also possibly be produced by additional energy sources, which are not included in Dowling and Ingersoll (1989), such as convection or other diabatic forcing. Such eddies might be a source of energy to both the unstable zonal flow and the large vortices.

Regarding the predictability of the emergence of the GRS-like vortex in the model, Dowling and Ingersoll (1989) showed that vastly different initial conditions lead to a very similar final form of the vortex. In other words, the GRS-like vortex is the only equilibrium state of the system. However, it is also found in Dowling and Ingersoll (1989) that the drift rate of a vortex depends sensitively on the bottom topography.

As mentioned above, the analog to the GRS is the hurricane in the atmosphere of the Earth. Like the GRS, a hurricane usually has a 100-to-1 ratio in the horizontal and vertical dimensions (although this ratio changes somewhat from case to case). However, unlike the GRS which is a persistent feature on Jupiter, a hurricane has a comparatively short existence. Unlike the GRS, hurricane genesis is highly sensitive to the initial disturbances, and, also dissimilar to the GRS, dissipates when this vortex moves to an environment in which its warm core structure cannot be maintained. The continued study of the similarities and differences in these two vortical structures need to be made.
XIV.11 Conclusions

This chapter presents an overview of a range of different atmospheric vortical structures ranging from small-scale turbulent features to large extratropical and tropical cyclones. Vortex features in the atmosphere have been investigated as three-dimensional features, and as two-dimensional, relatively long-lived structures that are predominately in the vertical or horizontal planes. The occurrence of latent heat release and the Coriolis effect complicates the study of vortical structure as contrasted with the more traditional fluid dynamics study of vortices.

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