On the Evaluation of the Mixing Layer During Elevated Plume Fumigation

Mordecay Segal and Roger A. Pielke
Colorado State University
Fort Collins, Colorado

The ground level concentrations of pollutants associated with the fumigation of elevated plumes during nocturnal inversion breakup is a frequent component in air quality evaluation reports. Conceptually, adequate evaluations of the spatial and temporal variations in the meteorological and concentration fields are needed in order to determine the peak concentration pattern during such processes, (see, e.g., Nieuwstadt and De Haan,[1] and Segal *et al.*[2]). In practice, less involved Gaussian oriented procedures, as outlined, e.g., by Turner[3] and Carpenter *et al.*,[4] are the most commonly adopted approaches. According to these procedures, a slice of the plume emitted when the inversion is eliminated at the stack height, h, is followed to the location where it is assumed to be vertically mixed within the mixing layer. At that location, which is determined mathematically by using the equation

$$\bar{h}_i = h + \delta h + 2.15\sigma_z, \tag{1}$$

the maximum concentrations due to fumigation are obtained. A crucial factor in this procedure is the evaluation of the time needed for the mixing layer to change depth from h to $\bar{h}_i$. The following evaluation for this time is given by Turner[3] and similarly by Carpenter *et al.*[4]

$$t_m = \frac{\bar{h}_i^2 - h^2}{4K} \tag{2}$$

where K is the diffusion coefficient relating to the capping stable layer. With regard to the values of K, Turner[3] refers to Hewson[5] who suggested $K = 3\ \mathrm{m^2 s^{-1}}$. Based on measured vertical profiles of temperature from the TVA experiment, Carpenter *et al.*[4] established a nomogram of K as dependent on the capping inversion potential temperature lapse rate, $\partial\theta^+/\partial z$. This nomogram is widely used in the evaluation of elevated plume fumigation during the nocturnal inversion breakup.

Physically, however, the rate of change of the depth of a mixing layer which is capped by inversion depends, in addition to the characteristics of the inversion, on the surface momentum and sensible heat fluxes and possibly on large scale subsidence vertical velocities. The sensible heat flux from the surface, for example, depends on such factors as the amount of solar radiation, temperature lapse within the surface layer, soil moisture and conductivity, and surface roughness, as well as long wave radiation from the surface. Therefore, for locations with different surface properties as well as solar radiation characteristics, the calculated values of K reported in Carpenter *et al.*[4] should not necessarily be appropriate.

It is the purpose of this note to suggest and discuss a more generalized and comprehensive procedure to evaluate t_m. This procedure, which is based on the formulations given in Tennekes[6] and Businger *et al.*,[7] is computationally reasonable and practical to use with regard to the needed data.

The Procedure

While examining recent literature relating to one-layer models of atmospheric entrainment (see Fig. 1 for a schematic illustration), it appears that considerable attention to the evaluation of the nocturnal surface inversion breakup has been given by Tennekes.[6] He suggests a set of two time dependent equations to determine the rate of nocturnal inversion breakup. Including synoptic subsidence, these equations can be written as

$$\Delta\left(\frac{\mathrm{d}h_i}{\mathrm{d}t} - w_i\right) = A \times \frac{T_0 u_*{}^3}{g h_i} + 0.2 S_0 \tag{3}$$

$$h_i \times \frac{\mathrm{d}\Delta}{\mathrm{d}t} + \Delta\left(\frac{\mathrm{d}h_i}{\mathrm{d}t} - w_i\right) = \frac{\partial\theta^+}{\partial z} \times h_i\left(\frac{\mathrm{d}h_i}{\mathrm{d}t} - w_i\right) - S_0 \tag{4}$$

where $S_0 = \rho c_p u_* \theta_*$ is the surface heat flux. The coefficient, A, for the mechanical entrainment term has been suggested by Tennekes as equal to 2.5, however, Driedonks[9] found that $A = 5$ is more appropriate.

Assuming that $\partial\theta^+/\partial z$ is given (e.g., through the measurement of the vertical profile of temperature immediately following sunrise from a radiosonde or tethersonde), the following information is needed in order to solve the set of Eq. 3 and 4:

i) Δ_0, namely the inversion strength when the inversion base is the stack top, h.
ii) The subsidence vertical velocity, w_i, (if pertinent).
iii) The friction velocity and temperature, u_* and θ_* as a function of time.

As discussed by Tennekes,[6] assuming $w_i = 0$ and $\mathrm{d}(\Delta h_i)/\mathrm{d}t = 0$, the following simplified equation can replace Eq. 3 and 4:

$$\frac{1}{2} \times \frac{\partial\theta^+}{\partial z}(\bar{h}_i^2 - h^2) = \int_0^{t_m} S_0(t) \times \mathrm{d}t \tag{5}$$

He estimates that using Eq. 5, regardless of whether the condition $\mathrm{d}(\Delta h_i)/\mathrm{d}t = 0$ is satisfied, may introduce errors of about 20% in the evaluation of h_i. As reported by Turner,[3] Eq. 5 has been suggested also by Pooler[8] for the evaluation of t_m in plume fumigation situations. Using Eq. 5, only u_* and θ_* as a function of time are needed.

In this note, a procedure to solve Eq. 3 and 4, or alternatively, Eq. 5, will be discussed by suggesting techniques to evaluate the variables listed above in i), ii), and iii).

The Initial Inversion Strength, Δ_0

The inversion strength Δ, which is the potential temperature difference across the inversion base (see Figure 1), might not always be well described when examining observational temperature profiles. However, within the assumptions of the model, Δ_0 can be mathematically evaluated if θ_m and the θ profile within the inversion are known, when the inversion base is at the stack top, h. Since during this time the surface layer is not too deep, temperature measurements at the edge of the surface layer can be easily performed for evaluation of θ_m.

The Vertical Synoptic Velocity, w_i

The vertical velocity w_i at the level h_i, can be approximated, using synoptic and if available mesoscale model outputs. The National Weather Service, for example, routinely outputs vertical velocity at 700 mb at 6-h time intervals using the Limited Fine Mesh (LFM) model and this information can be interpolated to the inversion height. On the synoptic scale, w_i may become relatively important, for example, when strong synoptic subsidence prevails. It is worth noting that in these cases, a merging of the nocturnal ground inversion and upper layer inversion is possible (e.g. Morgan and Bornstein[10]), thereby creating a deep inversion layer in the lower atmosphere.

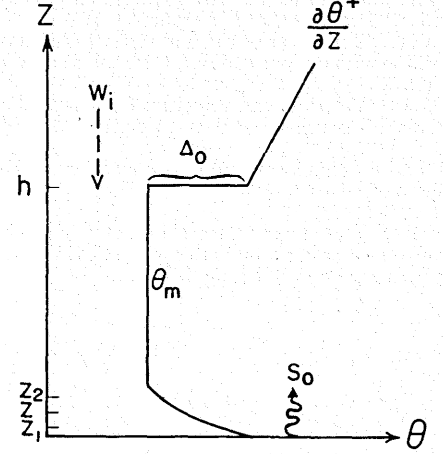

Figure 1. Schematic illustration of pertinent fumigation characteristics as the inversion breakup is at the stack height.

In studies of Tucker[11] and Banta[12] the evaluations of the vertical profile of w have been performed by utilizing sequential radiosonde wind and temperature profiles over a single site. Although this type of evaluation is applicable to the free atmosphere, where the adiabatic method of computing w can be most easily applied, utilizing it can provide some general indication as to the strength of the subsidence in the lower atmosphere.

The study of the intensity characteristic of w_i for certain sites, with relation to seasons and synoptic high pressure systems, can be obtained by adopting an inverse problem. Namely, a direct measurement can be used to determine the variables in Eq. 3 and 4 needed to compute w_i. Such an approach for evaluating w_i has been adapted by Johnson.[13] With this type of approach, characteristic values of w_i at specific geographic locations can be evaluated climatologically as a function of season and location relative to the major synoptic weather features (e.g., see Pielke[14]).

Friction Velocity and Temperature, u_* and θ_*

Using the expression $\xi = z/L$, where L is the Monin-Obukhov length, defined as $L = \theta u_*^2/k_0 g\theta_*$, and the expressions for the unstable surface layer nondimensional wind and potential temperature profiles according to Businger *et al.*,[7]

$$\phi_m = (1 - 15\,\xi)^{-1/4} \qquad (7)$$

$$\phi_H = 0.74\,(1 - 9\,\xi)^{-1/2}, \qquad (8)$$

u_* and θ_* are obtained from the integrated form of the profiles

$$u_* = k_0 u_z \Big/ \left[\ln\left(\frac{z}{z_0}\right) - \psi_1\right] \qquad (9)$$

$$\theta_* = k_0(\theta_z - \theta_{z_0})/[0.74 \ln(z/z_0) - \psi_2] \qquad (10)$$

$$\text{with} \quad \psi_1 = 2\ln[(1 + \phi_m^{-1})/2] + \ln[(1 + \phi_m^{-2})/2] - 2\tan^{-1}\phi_m^{-1} + \frac{\pi}{2} \qquad (11)$$

$$\text{and} \quad \psi_2 = 2\ln[(1 + 0.74\,\phi_H^{-1})/2] \qquad (12)$$

where the subscript z refers to a height within the surface layer, usually within the first few meters above the ground. Hence, if u_z and $\theta_z - \theta_{z_0}$ are given, an iterative solution of Eq. 9 and 10 provides u_* and θ_*.

For a specific location, one has to determine z_0 by using u_z from two levels during strong winds, i.e., a neutrally stratified surface layer where $u_z = (u_*/k_0)\ln(z/z_0)$. A value for z_0 can be estimated for non-neutral conditions providing z_0 is not a function of u_z, while u_z can be provided from wind speed measurements at several meters above the ground. With regard to $\theta_z - \theta_{z_0}$, obtaining observed values for θ_{z_0} may be practically difficult. However, having θ at two levels, z_1 and z_2 within the surface layer, Eq. 10 can be replaced by:

$$\theta_* = k_0(\theta_{z_2} - \theta_{z_1})/[0.74\ln(z_2/z_1) + \psi_H(z_1/L) - \psi_H(z_2/L)] \qquad (13)$$

Such type of data, namely, values for θ at two levels within the surface layer, is frequently available, and if not, can easily be obtained for sites of interest. It should be noted that the level at which the wind speed measurement is taken is not necessarily the same as either of the θ levels. In situations where profile measurements of u and θ are available, the evaluation of u_* and θ_* may be improved by using the best fit of these data to Eq. 9 and 10.

Obtaining the variables described above makes possible the evaluation of t_m through Eq. 3 and 4, or in the simple case, from Eq. 5, where only u_* and θ_* are needed. It is worth noting that while using these equations, it is assumed that the terrain is homogeneous and that horizontal advection of temperature,

c_p = specific heat of air
g = acceleration of gravity
h = stack height
h_i = mixing layer depth
$\hat{h}_i$ = mixing layer depth when peak fumigation concentrations are obtained
δh = plume elevation above the stack top
K = eddy diffusion coefficient
k_0 = von Karman's constant
S_0 = surface sensible heat flux
T_0 = reference temperature for the mixing layer
t_m = time in which fumigation occurs
u_z = horizontal velocity at height z
u_* = friction velocity
w = vertical components of the wind
w_i = synoptic vertical velocity at h_i
z_0 = roughness parameter
θ = potential temperature
θ_m = mixing layer potential temperature
θ_* = surface friction temperature
$\partial\theta^+/\partial z$ = potential temperature gradient immediately above the top of the mixing layer
Δ = inversion strength
Δ_0 = the inversion strength at the stack top
ρ = air density
σ_z = standard deviation of plume concentration in the vertical

moisture and velocity are insignificant relative to vertical turbulent mixing.

Finally, application of the procedure is mostly suggested for relatively high stacks such as those of power plants which are substantially above the surface layer, in order to avoid situations in which the dynamics of the surface layer dominates the entire planetary boundary layer. In addition, in these cases, the surface layer becomes reasonably stationary in relation to the time period of fumigation so that the use of the surface layer similarity formulations, as proposed by Businger *et al.*,[7] becomes conceptually adequate.

Summary and Conclusions

A procedure to evaluate the rate of change of the mixing layer, in order to estimate fumigation from an elevated plume during the nocturnal inversion breakup was suggested. The procedure is based on the current available parameterization of planetary boundary layer processes and it is more comprehensive and generalized as compared with the current procedures usually adopted in operational air quality studies. The needed observed meteorological data are temperature and wind within the surface layer depth during the fumigation process and only a single profile of the temperature above the stack top prior to the beginning of the process. Observationally oriented evaluations of the suggested procedure should be considered for further insight and refinements as to its application.

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Mr. Segal and Dr. Pielke are with the Department of Atmospheric Science, Colorado State University, Fort Collins, CO 80523. This note manuscript was peer reviewed.