Is Weather Chaotic? Coexistence of Chaos and Order within a Generalized Lorenz Model

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Is Weather Chaotic? Coexistence of Chaos and Order Within a Generalized Lorenz Model

by
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Big Data, Data Assimilation and Uncertainty Quantification  
Institut Henri Poincaré (IHP), Paris, France  
12-15 November 2019
Outline

- Introduction
  - 30 day Predictions of African Easterly Waves (AEWs) and Hurricanes
  - Goals and Approaches

- Lorenz Models (Lorenz, 1963, 1969)
  - Chaos and Two Kinds of Butterfly Effects (BE1 and BE2)
  - The Lorenz 1963 Model and BE1/Chaos
  - Three Types of Solutions (e.g., Steady-state, Chaotic, and Limit Cycle Orbits)
  - The Lorenz 1969 Model and BE2/Instability

- Major Features of Lorenz’s Butterfly
  - Divergence, Boundedness, and Recurrence

- A Generalized Lorenz Model (Shen, 2019a)
  - Slow and Fast Variables
  - Aggregated Nonlinear Negative Feedback
  - Two Kinds of Attractor Coexistence: Coexistence of Chaos and Order

- A Hypothetical Mechanism for Predictability of AEWs

- Summary and Outlook
Simulations of Helene (2006) Between Day 22-30

(Helene: 12-24 September, 2006)

- How can high-resolution global models have skill?

Goals and Approaches

Our goals include addressing the following questions:

- Can global models have skill for extended-range (15-30 day) numerical weather prediction? Why?
- Is weather chaotic?

To achieve our goals, we performed a comprehensive literature review and derived a generalized Lorenz model (GLM) to:

1. understand butterfly effects (i.e., chaos theory),
2. reveal and detect the coexistence of chaotic and non-chaotic processes,
3. emphasize the dual nature of chaos and order in weather, and
4. propose a hypothetical mechanism for the periodicity and predictability (of multiple African easterly waves, AEWs)
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Butterfly Effect of the First and Second Kind

Two kinds of butterfly effects can be identified as follows (Lorenz, 1963, 1972):

1. The butterfly effect of the first kind (BE1):
   Indicating *sensitive dependence on initial conditions* (Lorenz, 1963).
   - control run (blue): 
     \[ (X, Y, Z) = (0, 1, 0) \]
   - parallel run (red):
     \[ (X, Y, Z) = (0, 1 + \varepsilon, 0), \quad \varepsilon = 10 \]

2. The butterfly effect of the second kind (BE2):
   A metaphor (or symbol) for indicating that small perturbations can create a large-scale organized system (Lorenz, 1972/1969).
The Lorenz (1963) Model (3DLM)

The classical Lorenz model (Lorenz, 1963) with three variables and three parameters, referred to as the 3DLM, is written as follows:

\[
\frac{dX}{d\tau} = -\sigma X + \sigma Y, \\
\frac{dY}{d\tau} = -XZ + rX - Y, \\
\frac{dZ}{d\tau} = XY - bZ.
\]

- **r** – Rayleigh number: \((Ra/Rc)\) a dimensionless measure of the temperature difference between the top and bottom surfaces of a liquid; proportional to effective force on a fluid;
- **σ** – Prandtl number: \((\nu/\kappa)\) the ratio of the kinetic viscosity (κ, momentum diffusivity) to the thermal diffusivity (ν);
- **b** – Physical proportion: \((4/(1+a^2))\), \(b = 8/3\);
- **a** – \(a = l/m\), the ratio of the vertical height, \(h\), of the fluid layer to the horizontal size of the convection rolls. \(b = 8/3\); \(l = a\pi/H\) and \(m = \pi/H\).

- Note that \(X\), \(Y\), and \(Z\) represent the amplitudes of Fourier modes for the streamfunction and temperature.
- A **phase space** (or state space) is defined using the state variables \(X\), \(Y\) and \(Z\) as coordinates. The dimension of the phase space is determined by the number of variables.
- A **trajectory** or orbit is defined by time varying components within the phase space, also known as a solution.
- Two nonlinear terms form a nonlinear feedback loop (NFL).
Three Attractors Within the 3DLM

Depending on the relative strength of dissipations, four types of solutions within dissipative systems are:

a. Steady state solutions with a weak heating term (i.e., \( r < r_c; r_c = 24.74 \));
b. Chaotic solutions with a moderate heating term (i.e., \( r_c < r < R_c; R_c = 313 \ ));
c. Limit cycle solutions with a strong heating term (i.e., \( R_c < r \));
d. Coexistence of chaotic and steady-state solutions (24.06 < r < 24.74).

control run in blue
parallel run in red

A steady-state solution with a small r  A chaotic solution with a moderate r  A limit cycle with a large r
Three Attractors Within the 3DLM

A point attractor
(a spiral sink)

A chaotic attractor

A periodic attractor

A steady-state solution with a small $r$

A chaotic solution with a moderate $r$

A limit cycle with a large $r$
Limit Cycle: An Isolated Closed Orbit

- A limit cycle (black) is indicated by the convergence of 200 orbits (color).
- A limit cycle (LC) is an isolated closed orbit.
- Nearby trajectories spiral into it.
- LC orbits are determined by the structure of the system itself. It has no long term memory regarding ICs.

![Diagram](image)

- Color orbits: \( \tau \in [1,10] \)
- Black orbit: \( \tau \in [9,10] \)

![Graphs](image)

- Dependence of phases on ICs
- Oscillatory errors

A Dual Nature of Chaos and Order in Weather
Impact of Initial Tiny Perturbations Within the 3DLM

- Steady state or nonlinear periodic solutions have no (long-term) memory regarding their initial tiny perturbations
  - Initial tiny perturbations completely dissipate

- Chaotic solutions display a sensitive dependence on initial conditions
  - Initial tiny perturbations do not dissipate (before making a large impact)

- 3DLM: within the chaotic solutions, any tiny perturbation can cause large impacts. *Is this feature realistic?*

- We may ask what kind of impact tiny perturbations may introduce in real world models
Concurrent Visualizations: Butterfly Effects?

- A selected frame from a global animation of the vertical velocity in pressure coordinates from a run initialized at 0000 UTC 21 October 2005. The corresponding animation is available as a google document: [http://bit.ly/2GS2flD](http://bit.ly/2GS2flD). The animation displays dissipation of the initial noise associated with an imbalance between the model and the initial conditions (Shen, 2019b and references therein)
Lorenz 1963 and 1969 Models

- Lorenz (1963) Model (3DLM): ➔ BE1
  - nonlinear and chaotic
  - limited scale interactions (3 modes)
  - Lyapunov exponent (LE) analysis
  - KE and PE, PDE based (Rayleigh-Benard Convection)

  - multiscale but linear (21 modes)
  - growth rate analysis using a realistic basic state
  - KE, PDE based (a conservative system with no forcing or dissipation)

- Lorenz (1996/2005) Model:
  - nonlinear and chaotic with multiple spatial scales
  - equal weighting in dissipations
  - KE, not PDE based

2D (x,z) flow

\[
\frac{\partial}{\partial t} \nabla^2 \psi = -J(\psi, \nabla^2 \psi) + \nu \nabla^4 \psi + g \alpha \frac{\partial \theta}{\partial x},
\]

\[
\frac{\partial \theta}{\partial t} = -J(\psi, \theta) + \frac{\Delta T}{H} \frac{\partial \psi}{\partial x} + \kappa \nabla^2 \theta,
\]

Also see Rotunno and Synder (2008) and Durran and Gingrich (2014)

No PDEs

\[
\frac{dX_n}{d\tau} = -X_{n-2}X_{n-1} + X_{n-1}X_{n+1} - X_n + F;
\]
Comments on the Lorenz 1984 Model (Lorenz, 1990)

\[ \frac{dX}{dt} = -Y^2 - Z^2 - aX + aF, \]
\[ \frac{dY}{dt} = XY - bXZ - Y + G, \]
\[ \frac{dZ}{dt} = bXY + XZ - Z. \]

- The variable X represents the strength of a large scale westerly-wind current, and also the geostrophically equivalent large-scale poleward temperature gradient;

- Y and Z are the strengths of the cosine and sine phases of a chain of superposed waves, respectively.

The above idealized system was proposed by Lorenz in 1984 for qualitatively depicting atmospheric circulation, known as the Lorenz (1984) model. Due to the following issues, results obtained using the Lorenz 1984 model should be analyzed and interpreted with caution:

1. **Detailed derivations of the Lorenz (1984) model were missing** (e.g., Veen 2002a, b); it is difficult to trace the physical source of the forcing terms (parameters “F” and “G” in Eqs. (1)-(3) of Lorenz 1984) in the model.

2. As compared to fully dissipative systems where the time change rate of volume of the solutions is negative, **the volume of the solution** within the 1984 model **does not necessarily shrink to zero** (e.g., p. 380 of Lorenz 1990).
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- **Summary and Outlook**
What Lorenz’s Butterfly Really Reveals

The statement of “Orbits initially diverge and then curve back” includes the following major features of butterfly solutions:

1. **Divergence** of Trajectories

2. **Boundedness**
   - No “blow-up” solutions

3. **Recurrence/Folding**
   - Complex eigenvalues, $\lambda = \alpha + i\beta$: real part leads to a growing or decaying solution; imaginary part gives the oscillatory component.

4. **Error Saturations**
   - Max errors determined by the “size” of the butterfly’s wings

5. **Ergodicity** (Hilborn, 2000)
   - Time averages are the same as state space averages.

Mathematically, the butterfly solution can be represented as:

$$X(t) = e^{\alpha t} (\cos(\beta t) + i \sin(\beta t))$$

The solution grows or decays at an exponential rate.
Based on the previous discussions, we may ask whether the following folklore is an “accurate” analogy of the butterfly effect (Gleick, 1987; Drazin, 1992):

“For want of a nail, the shoe was lost. 
For want of a shoe, the horse was lost. 
For want of a horse, the rider was lost. 
For want of a rider, the battle was lost. 
For want of a battle, the kingdom was lost. 
And all for the want of a horseshoe nail.”

Do we all agree on the above? Prof. Lorenz expressed his concerns in 2008.

However, Lorenz (2008) made the following comments:

1. Let me say right now that I do not feel that this verse is describing true chaos, but better illustrates the simpler phenomenon of instability.
2. The implication is that subsequent small events will not reverse the outcome.

Lorenz’s comments support the view that the verse neither describes (local) time-varying convergence of trajectories nor indicates recurrence.
The Lorenz Model (Lorenz, 1963)

\[
\begin{align*}
\frac{dX}{d\tau} &= -\sigma X + \sigma Y, \\
\frac{dY}{d\tau} &= -XZ + rX - Y, \\
\frac{dZ}{d\tau} &= XY - bZ.
\end{align*}
\]

The Linear Geometric Model by (Guckenheimer and Williams, 1979)

\[
\begin{align*}
x' &= -3x, \\
y' &= 2y, \\
z' &= -z.
\end{align*}
\]

Importance of the Saddle Point

The Limiting Equations (Sparrow, 1982)

\[
\begin{align*}
\frac{dX}{d\tau} &= \sigma Y, \\
\frac{dY}{d\tau} &= -XZ \\
\frac{dZ}{d\tau} &= XY.
\end{align*}
\]

The Nonlinear Non-dissipative Lorenz Model (Shen, 2018)

\[
\begin{align*}
\frac{dX}{d\tau} &= \sigma Y, \\
\frac{dY}{d\tau} &= -XZ + rX, \\
\frac{dZ}{d\tau} &= XY.
\end{align*}
\]

Role of the NFL in producing oscillatory solutions

Role of the NFL in producing homoclinic orbits, as well as periodic solutions
The Role of Nonlinearity

- Within the 3D Nondissipative Lorenz model, which is a conservative system:
  - nonlinearity plays a role as a restoring force;
  - the upper half of the homoclinic orbit before it reaches its maximum can be depicted by the logistic equation;
  - time varying growth rates indicate energy conversion between kinetic energy and potential energy.

- Within the Logistic Equation (a.k.a. the error growth model):
  - nonlinearity suppresses growth rates;
  - the Logistic equation that is a first order ODE with real coefficients yields solutions with non-negative growth (or decay) rates during the entire lifetime.

- Within a 2D limit cycle model:
  - nonlinearity acts as a damping term;
  - evolution of the solution amplitude (instead of phase) can be depicted with a logistic equation.
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A Generalized Lorenz Model (GLM)

As discussed in Shen (2019a) and Shen et al. (2019), the GLM with many M modes possesses the following features:

1. any odd number of M greater than three; a conservative system in the dissipationless limit;

2. three types of solutions (that also appear within the 3DLM);

3. energy transfer across scales by the nonlinear feedback loop (NFL);

4. slow and fast variables across various scales;

5. aggregated negative feedback;

6. hierarchical scale dependence;

7. two kinds of attractor coexistence;
   - The 1\textsuperscript{st} kind of Coexistence for Chaotic and Steady-state Solutions
   - The 2\textsuperscript{nd} kind of Coexistence for Limit Cycle and Steady-state Solutions.
A Generalized Lorenz Model (GLM)

The GLM is derived based on extensions of the NFL that can provide negative nonlinear feedback to stabilize solutions. The GLM is written as follows:

\[
\begin{align*}
\frac{dX}{d\tau} &= -\sigma X + \sigma Y, \\
\frac{dY}{d\tau} &= -XZ + rX - Y, \\
\frac{dZ}{d\tau} &= XY - bZ,
\end{align*}
\]

• The “backbone” of the linearized NFL is analogous to the spring of the above system.
• A new pair of high wavenumber modes \((Y_j, Z_j)\) that extends the NFL creates an additional frequency in a new subsystem with a different spring constant.
Slow and Fast Variables Within the GLM

The following two equations suggest that $Y_j$ is a fast variable while $Y$ is a slow variable when $1/j$ is small (i.e., $j$ is large).

\[
\frac{dY}{d\tau} = -XZ + rX - Y \quad (2)
\]

\[
\frac{1}{j} \frac{dY_j}{d\tau} = \left( XZ_{j-1} - \frac{(j + 1)}{j} XZ_j - \frac{d_{j-1}}{j} Y_j \right) \quad j \in \mathbb{Z} : j \in [1, N] \quad (4)
\]

Note that the GLM is coupled by the extension of the nonlinear feedback loop and the coefficients of the terms on the right-hand sides continuously increase in association with increasing inclusion of high wavenumber modes, as shown in Eq. (4).
An Indicator of Aggregated Negative Feedback

A comparison of the 3D, 5D, 7D and 9D LMs, requiring larger heating parameters for the onset of chaos in higher-dimensional LMs, indicates aggregated negative feedback by small scale modes.

<table>
<thead>
<tr>
<th>model</th>
<th>( r_c )</th>
<th>heating terms</th>
<th>solutions</th>
<th>references</th>
</tr>
</thead>
<tbody>
<tr>
<td>3DLM</td>
<td>24.74</td>
<td>( r_X )</td>
<td>steady, chaotic, or LC</td>
<td>Lorenz (1963)</td>
</tr>
<tr>
<td>3D-NLM</td>
<td>n/a</td>
<td>( r_X )</td>
<td>periodic</td>
<td>Shen (2018)</td>
</tr>
<tr>
<td>5DLM</td>
<td>42.9</td>
<td>( r_X )</td>
<td>steady, chaotic, or LC/LT</td>
<td>Shen (2014a,2015a,b)</td>
</tr>
<tr>
<td>5D-NLM</td>
<td>n/a</td>
<td>( r_X )</td>
<td>quasi-periodic</td>
<td>Faghih-Naini and Shen (2018)</td>
</tr>
<tr>
<td>6DLM</td>
<td>41.1</td>
<td>( r_X, r_X_1 )</td>
<td>steady or chaotic</td>
<td>Shen (2015a,b)</td>
</tr>
<tr>
<td>7DLM</td>
<td>116.9</td>
<td>( r_X )</td>
<td>steady, chaotic or LC/LT</td>
<td>Shen (2016, 2017)</td>
</tr>
<tr>
<td>7D-NLM</td>
<td>n/a</td>
<td>( r_X )</td>
<td>quasi-periodic</td>
<td>Shen and Faghih-Naini (2017)</td>
</tr>
<tr>
<td>8DLM</td>
<td>103.4</td>
<td>( r_X, r_X_1 )</td>
<td>steady or chaotic</td>
<td>Shen (2017)</td>
</tr>
<tr>
<td>9DLM</td>
<td>102.9</td>
<td>( r_X, r_X_1, r_X_2 )</td>
<td>steady or chaotic</td>
<td>Shen (2017)</td>
</tr>
<tr>
<td>9DLMr</td>
<td>679.8</td>
<td>( r_X )</td>
<td>steady, chaotic, or LC/LT</td>
<td>Shen (2019a)</td>
</tr>
</tbody>
</table>

\( r_c \): a critical value of the Raleigh parameter for the onset of chaos; LC: limit cycle; LT: limit torus
The First Kind of Attractor Coexistence Within the 9DLM

For the 1st kind of attractor coexistence within the 9DLM, the appearance of a steady state solution (left and middle) or a chaotic solution (right) depends on the initial conditions. This indicates final state sensitivity to ICs.

non-chaotic orbits

chaotic orbit
The Second Kind of Attractor Coexistence Within the 9DLM

- A limit cycle (LC) is an isolated closed orbit.
- Nearby trajectories spiral into it.
- LC orbits are determined by the structure of the system itself.
  - The total simulation time is $\tau = 3.5$.
  - Transient orbits are only kept for the last 0.25 time units, i.e. for the time interval of $[\max (0, T-0.25), T]$ at a given time $T$.
  - The zoom-in of the domain starts at $\tau = 0.25$ and ends at $\tau = 0.45$, leading to a smooth domain change from $(X, Y_3, Z_3) = (-1300,1200) \times (-1100,1100) \times (-1000,1700)$ to $(-300;200) \times (-100,100) \times (0,700)$.

Figure: Time evolution of 2,048 orbits in the $X-Y_3-Z_3$ space using the 9DLM, showing spiral sinks and a limit cycle/torus solution. The animation is available from https://goo.gl/sMhoUb.
Two Kinds of Dependence on ICs

The 9DLM with attractor coexistence reveals:

- **final state sensitivity to ICs**, i.e., ICs determine whether solutions are chaotic or steady state;
- **sensitive dependence on ICs** for chaotic solutions; and
- **no long term memory of ICs** for steady solutions.

The role of initial tiny perturbations:

- Within attractor coexistence of the 9DLM, tiny perturbations may **dissipate completely or cause a large impact**, depending on various kinds of orbits (i.e., various basins of attraction).
- Within chaotic solutions of the 3DLM, any tiny perturbation can cause large impacts.

https://doi.org/10.1142/S0218127419500378
Coexisting Attractors in Ensemble Runs

Ensemble runs with \( N \) ICs distributed over a hypersphere centered at a non-trivial critical point with a radius of \( R \) (i.e., the spatial scale of ICs)

![Coexisting Attractors in Ensemble Runs](image)

Dependence of numerical simulations for chaotic and non-chaotic orbits on initial conditions (ICs). (a) 4,096 ICs distributed over a hypersphere with a radius of \( R = 5 \); (b) 4,096 ICs with a \( R = 100 \); (c) 512 ICs with a \( R = 200 \); (d) 128 ICs with a \( R = 200 \); (e) 128 ICs with a \( R = 300 \); and (f) 64 ICs with a \( R = 500 \).
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Oscillatory Forecast Scores in the 30 Day Run: Why?

Correlation Coefficients (CCs)

Is the forecast score a monotonically decreasing function of time?

Note that a limit cycle is oscillatory and two chaotic orbits may produce time varying convergence and divergence.
A Hypothetical Mechanism for the Predictability at Extended-Range Scales

<table>
<thead>
<tr>
<th>Limit Cycle</th>
<th>African Easterly Waves</th>
</tr>
</thead>
<tbody>
<tr>
<td>features</td>
<td>periodic</td>
</tr>
<tr>
<td>errors</td>
<td>oscillatory</td>
</tr>
<tr>
<td>system conditions</td>
<td>strong heating + nonlinearity</td>
</tr>
</tbody>
</table>

1. The model displayed an ability to simulate the initiation of multiple (4+) AEWs.
2. The model was able to simulate downscale transfer from a specific AEW (e.g., the 4th one) to a system at a smaller scale (e.g., TC).
3. The model capability in (2) and (3) may lead to a predictability of (20 + 3) days.

A Dual Nature of Chaos and Order in Weather

IHP, Paris, France, 12 Nov. 2019

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Additional Support for Oscillatory Components

- Based on dishpan experiments (e.g., Fultz et al. 1959; Hide 1953), Lorenz (1993) suggested three types of solutions, including: (1) steady state solutions, (2) irregular chaotic solutions, and (3) vacillation. “Amplitude vacillation” is defined as a solution whose amplitude grows and periodically decays in a regular cycle (Lorenz 1963c; Ghil and Childress 1987; Ghil et al. 2010). Studies by Pedlosky and Smith (e.g., Pedlosky 1972; Smith 1975; Smith and Reilly 1977) found that amplitude vacillation can be viewed as a limit cycle solution.

- Lorenz (1990) applied the Lorenz (1984) model to reveal “chaotic winter and non-chaotic summer” (in the bottom left figure).

- Using the NCAR WACCM3 (Whole Atmosphere Community Climate Model Model), Liu et al. (2009) documented oscillatory root mean square (RMS) errors (i.e., with no error saturation) (in the bottom right figure).

- \[ t = 0 \quad 3 \quad 6 \quad 9 \quad 12 \quad (Mo) \quad 0 \quad 30 \quad 60 \quad 90 \quad 100 \quad (days) \]
Concluding Remarks

1. Two kinds of butterfly effects in Lorenz studies can be defined as follows:
   • The **BE1**: the sensitive dependence of solutions on initial conditions;
   • The **BE2**: a metaphor for indicating the enabling role of a tiny perturbation in producing an organized large-scale system.

2. The GLM possesses the following features:
   a) Three types of attractors; b) Two kinds of attractor coexistence; c) Aggregated negative feedback; d) Hierarchical scale dependence.

3. Chaotic solutions only appear within the finite range of the Rayleigh parameters. Chaotic and non-chaotic orbits may coexist, displaying two kinds of data dependence. The BE1 does not always appear.

4. “As with Poincare and Birkhoff, everything centers around periodic solutions,” Lorenz and chaos advocates focused on the existence of non-periodic solutions and their complexities.

5. We propose that the entirety of weather possesses a dual nature of chaos and order associated with chaotic and non-chaotic processes, respectively. We emphasize the importance of taking into consideration the duality of solutions to revisit the predictability problem.
Takeaway Messages

- The entirety of weather possesses a dual nature of chaos and order.
  - The above refined view is neither too optimistic nor pessimistic as compared to the Laplacian view of deterministic predictability and the Lorenz view of deterministic chaos.

- "there is no reason that the limit of predictability is a fixed number" as suggested by Prof. Arakawa (Lewis, 2005, MWR).
  - In some cases, we obtained realistic predictions with a predictability of over two weeks.
Acknowledgments and References

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Selected References: